1) The DC/DC converter shown in Fig. 1 operates in CCM. The switches are in position 1 for $0 < t < DT_s$ and in position 2 for $DT_s < t < T_s$. Assume that all components are ideal.
a) Derive the expression of V_O/V_{IN} as a function of D and sketch its plot.

$L(V_S - V_O)D + L(V_S + V_O)(1-D) = 0$

$V_S \cancel{D} - V_O D + V_S - \cancel{V_S D} + V_O - V_O D = 0$

$V_S = (2D-1)V_O$

$\frac{1}{2D-1} = \frac{V_O}{V_S}$

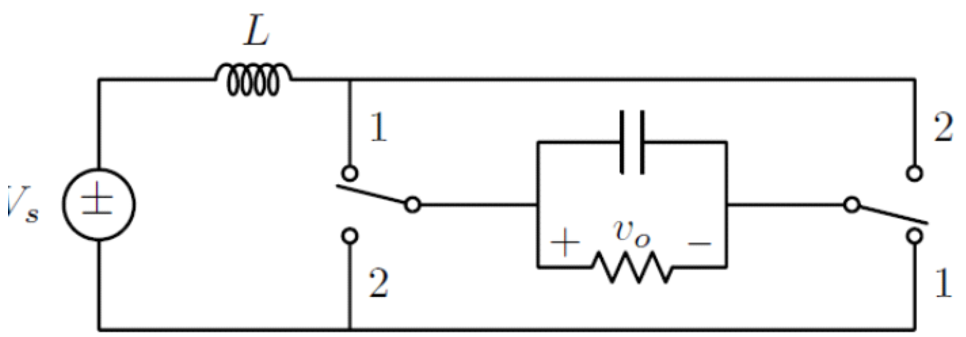


Fig. 1

Assuming $V_{IN} = 2.5V$, $D = 0.75$, $R = 5\Omega$, $f_{sw} = 1.5$ MHz:

b) Calculate the output voltage, V_O.

$\frac{1}{2\cdot 0.75 - 1}\cdot 2.5 = V_o = 5V$

c) Select L such that the peak-to-peak current ripple is 30% of the nominal current.

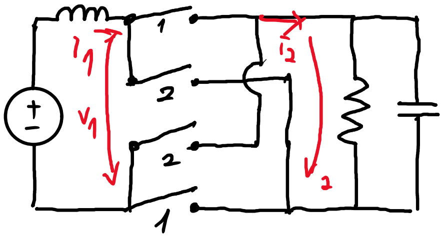
$I_L = I_{in}$ $\quad P_{in} = P_o = V_i I_i = \frac{V_o^2}{R} \Rightarrow I_L = \frac{V_o^2}{V_i R} = 2A$

$V = L\frac{\Delta i}{\Delta t}$ $\quad \frac{(V_S - V_O)DT_S}{L} \le 0.3 I_L \Rightarrow \frac{|V_S - V_O| DT_S}{0.3 I_L} \le L \Rightarrow L \geq 2.083\mu H$

d) Calculate the peak-to-peak output voltage ripple (the ESR can be neglected).

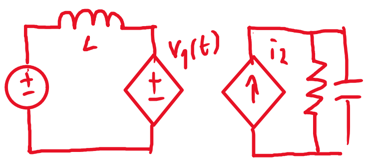
$I = C\frac{\Delta V}{\Delta t} \Rightarrow \frac{(I_i - I_o)DT_S}{C} = \Delta V = \dots ?$

e) Show that the converter of Fig. 1 can be drawn in the electrically identical form shown in Fig. 2 . Sketch the waveforms $v_1(t)$ and $i_2(t)$. Use the circuit averaging method to derive a large-signal averaged model for this converter.



$i_2(t) = i_1 q(t) - i_1(1-q(t)) = i_1(2q(t)-1)$

$v_1(t) = v_2 q(t) - v_2(1-q(t)) = v_2(2q(t)-1)$



$\bar{v}_1(t) \cong \bar{v}_2(t)(2d(t)-1)$

$\bar{i}_2(t) \cong \bar{i}_1(t)(2d(t)-1)$

f) Perturb and linearize the averaged circuit model to obtain a small-signal ac equivalent circuit of the converter.

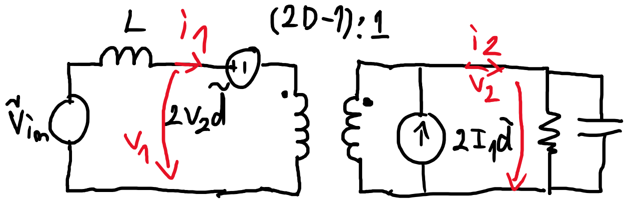
g) Derive the transfer function $G_{oc}(s) = \frac{\tilde{v}_o}{\tilde{v}_c}$ (assume G_{PWM}=1 V^{-1}). Sketch the Bode plot of $|G_{oc}(j\omega)|$.

Perturb

$$V_1+\tilde{v}_1 = (V_2+\tilde{v}_2)(2(D+\tilde{d})-1) \qquad V_1+\tilde{v}_1 = V_2(2D-1)+2V_2\tilde{d}+\tilde{v}_2(2D-1)$$

$$I_2+\tilde{i}_2 = (I_1+\tilde{i}_1)(2(D+\tilde{d})-1) \qquad I_2+\tilde{i}_2 = I_1(2D-1)+2I_1\tilde{d}+\tilde{i}_1(2D-1)$$

$$DC\begin{cases} V_1 = V_2(2D-1) \\ I_2 = I_1(2D-1)\end{cases} \qquad AC\begin{cases} \tilde{v}_1 = 2V_2\tilde{d}+\tilde{v}_2(2D-1) \\ \tilde{i}_2 = 2I_1\tilde{d}+\tilde{i}_1(2D-1)\end{cases}$$



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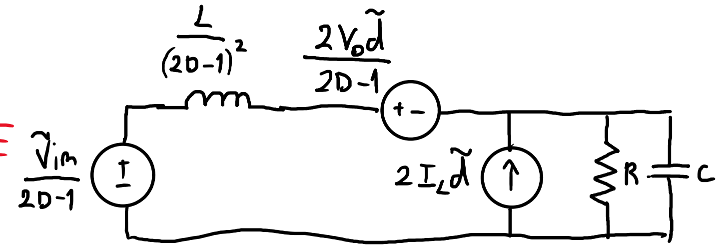
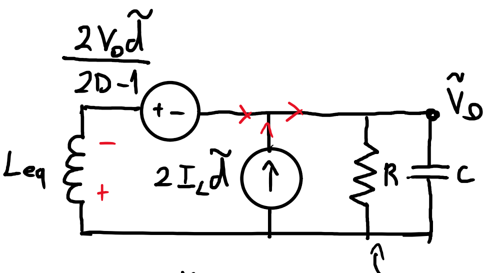


$I_1 = I_L \quad V_2 = V_0$

$$G_{oc} = \frac{\tilde{v}_o(s)}{\tilde{v}_c(s)} = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}\cdot\underbrace{\frac{\tilde{d}(s)}{\tilde{v}_c(s)}}_{G_{PWM}=1} = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

$\tilde{v}_{in} = 0$

$L_{eq} = \frac{L}{(2D-1)^2}$



$I_L = \frac{V_o}{R(2D-1)} \qquad \left(\frac{1}{R}+sC\right)^{-1} = \frac{R}{1+sCR}$

$$\tilde{v}_o = -\left(sL_{eq}\left(\frac{\tilde{v}_o(1+sCR)}{R}-2I_L\tilde{d}\right)+\frac{2V_o\tilde{d}}{2D-1}\right) \Leftrightarrow$$

$$\Leftrightarrow \tilde{v}_o = -\tilde{v}_o\cdot\frac{sL_{eq}(1+sCR)}{R}+s2L_{eq}I_L\tilde{d}-\frac{2V_o}{2D-1}\tilde{d} \Leftrightarrow$$

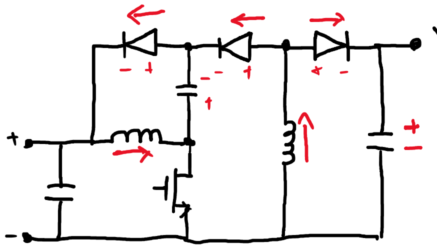
$$\Leftrightarrow \tilde{v}_o\left(1+\frac{sL_{eq}(1+sCR)}{R}\right) = \tilde{d}\frac{2V_o}{2D-1}\left(s\frac{L_{eq}}{R}-1\right) \Leftrightarrow$$

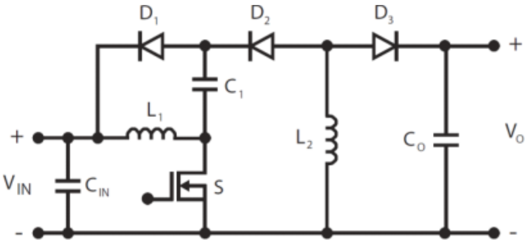
$$\Leftrightarrow \frac{\tilde{v}_o}{\tilde{d}} = \frac{2V_o}{2D-1}\frac{s\frac{L_{eq}}{R}-1}{1+\frac{sL_{eq}(1+sCR)}{R}} = \frac{2V_o}{2D-1}\cdot\frac{s\frac{L_{eq}}{R}-1}{s^2L_{eq}C+s\frac{L_{eq}}{R}+1}$$

$$L_{eq} = \frac{L}{2D-1} \quad \zeta = \frac{1}{2R}\sqrt{\frac{L_{eq}}{C}} \qquad \frac{\tilde{v}_o}{\tilde{d}} = \frac{2V_o}{2D-1}\cdot\frac{\frac{1}{\omega_{RHPZ}}-1}{\frac{s^2}{\omega_m^2}+\frac{2\zeta}{\omega_m}s+1}$$

$$\omega_m = \frac{1}{\sqrt{L_{eq}C}} \quad \omega_{RHPZ} = \frac{R}{L_{eq}}$$

2) The DC-DC converter shown in Fig. 3 operates in CCM at switching frequency f_{sw} = 500 kHz.
a) Draw a plot of the DC voltage transfer function V_O/V_{IN} as a function of the duty cycle D.



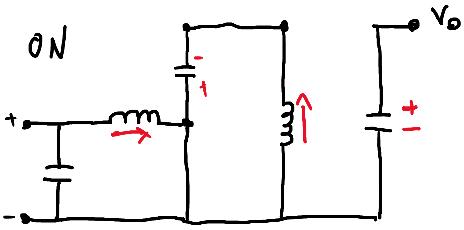


$$\begin{cases} V_i D - V_C(1-D) = 0 \\ -V_C D + V_O(1-D) = 0 \end{cases}$$

$$V_C = \frac{V_O(1-D)}{D}$$

$$V_i D - \frac{V_O(1-D)^2}{D} = 0$$

ON



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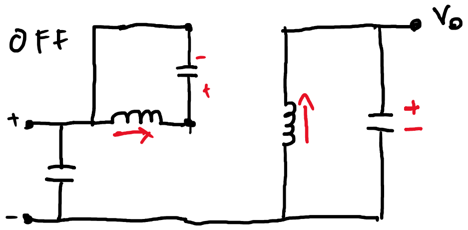

$$\frac{V_O}{V_i} = \frac{D^2}{(1-D)^2}$$

Given the following specifications: V_{IN} = 24 V, V_O = 1.5 V, P_O = 6 W
b) Calculate the duty-cycle D.
c) Calculate the average inductor currents.

$$\frac{V_O}{V_{in}} = \left(\frac{D}{(1-D)}\right)^2 \Rightarrow \frac{1}{4} = \frac{D}{1-D} \Rightarrow D = 0.20$$

$$-I_{L_2} D + I_{L_1}(1-D) = 0 \leftarrow \text{charge balance}$$

$$I_{L_1} = \frac{I_2 D}{1-D} = 1.25A$$

$$I_O = 4A$$

$$I_O = I_{L_2} \cdot (1-D) \Leftrightarrow \frac{6}{1.5 \cdot 0.8} = I_{L_2} = 5A$$

d) Select L_1 and L_2 such that the peak-to-peak current ripple is less than 25% of the average current.

$$V = L\frac{\Delta i}{\Delta t} \qquad \frac{V \Delta t}{L} \leqslant \Delta i$$

$$L_1: \quad \frac{V_i D T_s}{L_1} \leqslant 0.25 \cdot 1.25 \Rightarrow L_1 \geqslant \frac{24 \cdot 0.2 \cdot 1/500k}{0.25 \cdot 1.25} \Leftrightarrow L_1 \geqslant 30.7\mu H$$

$$L_2: \quad \frac{V_O(1-D)T_s}{L_2} \leqslant 0.25 \cdot I_{L_2} \Rightarrow L_2 \geqslant \frac{1.5 \cdot 0.8}{500k \cdot 0.25 \cdot 5} \Leftrightarrow L_2 \geqslant 1.92\mu H$$

e) Select the capacitor C_1 such that the ripple voltage (peak-to-peak) is less than 1V.

$$I = C\frac{\Delta V}{\Delta t} \Rightarrow C > \frac{I_{L_1}(1-D)T_s}{\Delta V} \Rightarrow C > 1.25 \cdot 0.8 \cdot 1/500k \Rightarrow C > 2\mu F$$

f) Derive expressions for L_{crit1} and L_{crit2} as a function of D.

$$\frac{V \Delta t}{L} = \Delta i \qquad L_1: \frac{V_i D T_s}{L_{c1}} = 2 I_{L_1} \Rightarrow \frac{V_O \frac{(1-D)^2}{D^{\cancel{2}}} \cancel{D} T_s}{2 \cdot \frac{I_O D}{(1-D)^2}} = L_{c1} \Leftrightarrow L_{c1} = \frac{R_O T_s}{2} \frac{(1-D)^4}{D^2}$$

$$L_2: \frac{V_O(1-D)T_s}{L_{c2}} = 2 I_{L_2} \Rightarrow L_{c2} = \frac{V_O}{I_O} \cdot \frac{(1-D)^2 T_s}{2} = \frac{R_O T_s}{2}(1-D)^2$$

g) Does the diode D_2 set a limit to the maximum duty-cycle? Briefly motivate your answer. Yes it cannot boost
$D_{max} = 0.5$