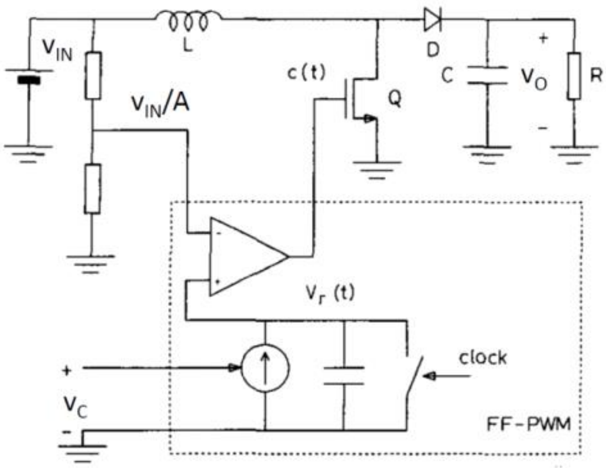
1) A boost converter with feedforward pulse-width modulator is shown in Fig 1. The converter operates in CCM. A sawtooth waveform $v_r(t)$ is obtained using an integrator with reset. A clock signal resets the integrator periodically with period T_s. The capacitor charging current in the integrator is proportional to the slowly-varying modulating input v_c. The periodic sawtooth waveform $v_r(t)$ is given by

$$v_r(t) = v_c \cdot \frac{t}{T_s}; \qquad v_r(t + kT_s) = v_r(t)$$

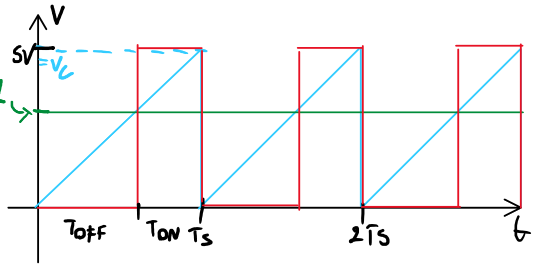


$v_r(t)$ is compared with the scaled input voltage v_{IN}/A. The output of the comparator is the pulsating signal c(t) that controls the switching transistor Q.
Given the following specifications: V_{IN} = 35V, V_O = 50 V, f_s = 100 kHz, V_C = 5V:
a) draw a plot of the voltage waveforms at the input and output of the comparator.

a) $V^+ = V_r$

$V^- = \frac{V_{IN}}{A}$

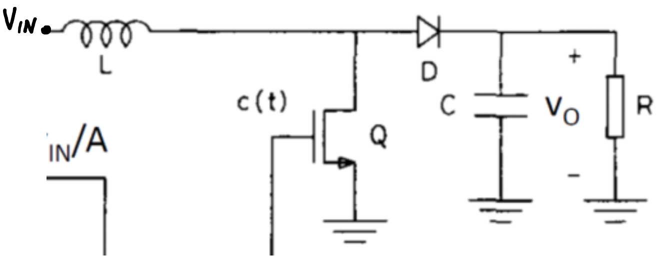
$V_o = c$



b) Derive the expression of D as a function of V_{IN} and V_C.

c) Derive the expression of the DC voltage transfer function V_O/V_C.

$$\frac{T_{OFF}}{V_{IN}/A} = \frac{T_S}{V_C} \Rightarrow 1-D = \frac{V_{IN}/A}{V_C} \Leftrightarrow D = 1 - \frac{V_{IN}}{A \cdot V_C}$$



$$V_{IN} D + (V_{IN} - V_O)(1-D) = 0 \Leftrightarrow V_{IN} D + V_{IN} - V_{IN} D - V_O(1-D) = 0$$

$$\frac{V_O}{V_{IN}} = \frac{1}{1-D} \qquad V_{IN} = (1-D) V_C A$$

$$\frac{V_O}{(1-D) V_C A} = \frac{1}{1-D} \Leftrightarrow \frac{V_O}{V_C} = A$$

d) Calculate the voltage divider ratio A. $A = \frac{V_O}{V_C} = \frac{50}{5} = 10$

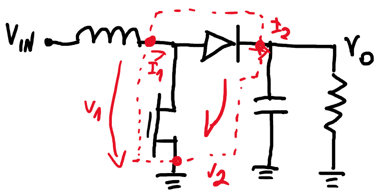
2a) Derive the line-to-output (audiosusceptibility) and the control-to-duty cycle transfer functions for a standard boost converter (no feedforward).

Apply small perturbations $\tilde{v}_c$ and $\tilde{v}_{IN}$ to V_C and V_{IN} respectively.

b) Calculate $\tilde{d}$ as a function of $\tilde{v}_c$ and $\tilde{v}_{IN}$.

c) Derive the control-to-output transfer transfer function $G_{OC_{F.F.}}(s) = \frac{\tilde{v}_O(s)}{\tilde{v}_C(s)}$.

d) Derive the line-to-output transfer function $G_{OL_{F.F.}}(s) = \frac{\tilde{v}_O(s)}{\tilde{v}_{IN}(s)}$.



$$V_1(t) = V_2(1 - q(t)) \qquad I_2(t) = I_1(1 - q(t)) \Rightarrow Avg. \Rightarrow \langle V_1 \rangle \cong \langle V_2 \rangle (1 - d(t)) \qquad \langle I_2 \rangle \cong \langle I_1 \rangle (1 - d(t))$$

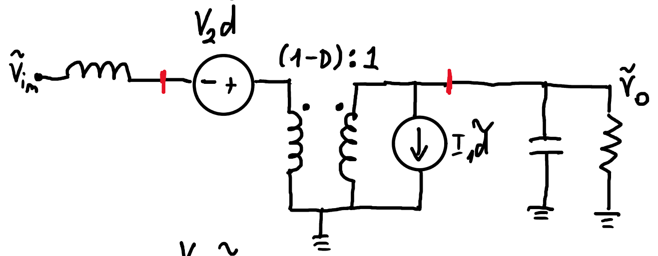
Pert

$$V_1 + \tilde{v}_1 = (V_2 + \tilde{v}_2)(1 - D - \tilde{d}) \Rightarrow \begin{cases} V_1 = V_2(1-D) \\ \tilde{v}_1 = -V_2 \tilde{d} + \tilde{v}_2(1-D) \end{cases}$$

$$I_2 + \tilde{i}_2 = (I_1 + \tilde{i}_1)(1 - D - \tilde{d}) \Rightarrow \begin{cases} I_2 = I_1(1-D) \\ \tilde{i}_2 = -I_1 \tilde{d} + \tilde{i}_1(1-D) \end{cases}$$

$$\begin{cases} V_1 = V_2(1-D) \\ \tilde{v}_1 = -V_2\tilde{d} + \tilde{v}_2(1-D) \end{cases}$$

$$\begin{cases} I_2 = I_1(1-D) \\ \tilde{i}_2 = -I_1\tilde{d} + \tilde{i}_1(1-D) \end{cases}$$

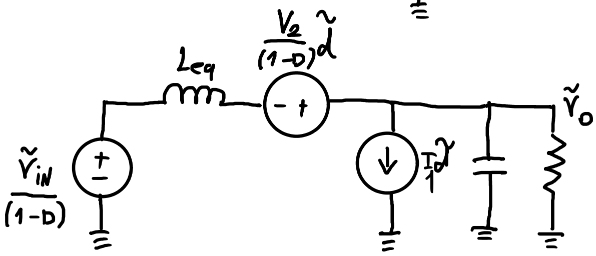


$$\frac{V_O}{V_{IN}} = \frac{1}{1-D}$$

$$V_2 = V_o$$

$$I_1 = I_{in} = I_o\left(\frac{1}{1-D}\right)$$

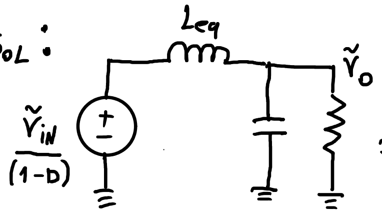
$$L_{eq} = \frac{L}{(1-D)^2}$$



$$G_{od} = \left.\frac{\tilde{v}_o}{\tilde{d}}\right|_{\tilde{v}_{in}=0,\ \tilde{i}_{im}=0}$$

$$G_{ol} = \left.\frac{\tilde{v}_o}{\tilde{v}_{in}}\right|_{\tilde{d}=0}$$

G_{oL}:



$$R//C = \left(\frac{1}{R} + j\omega C\right)^{-1} = \frac{R}{1+sRC}$$

$$\tilde{v}_o = \frac{\frac{R}{1+sRC}}{\frac{R}{1+sRC} + sL_{eq}} \cdot \frac{\tilde{v}_{im}}{(1-D)} \Leftrightarrow \frac{\tilde{v}_o}{\tilde{v}_{im}} = \frac{1}{1-D} \cdot \frac{R}{R + sL_{eq}(1+sRC)} =$$

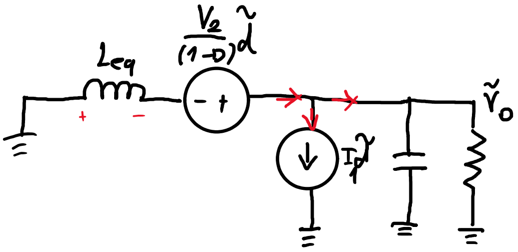
$$= \frac{1}{(1-D)} \cdot \frac{1}{1 + s\frac{L_{eq}}{R} + s^2 C L_{eq}} = \frac{1}{1-D} \cdot \frac{1}{1 + \frac{1}{Q}\frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}}$$

$$\omega_0 = \frac{1}{\sqrt{C L_{eq}}}$$

$$\frac{1}{Q}\cdot\frac{1}{\omega_0} = \frac{L_{eq}}{R} \Rightarrow Q = R\sqrt{\frac{C}{L_{eq}}}$$

$$L_{eq} = \frac{L}{(1-D)^2}$$

G_{oD}:



$$G_{oD} = \frac{\tilde{v}_o}{\tilde{d}}$$

$$I_1 = I_{IN} = \frac{V_{out}}{R}\cdot\frac{1}{1-D}$$

$$V_2 = V_{out}$$

$$\tilde{v}_o \cdot \frac{1+sRC}{R} + \frac{V_{out}}{R}\cdot\frac{1}{1-D}\tilde{d} = \frac{\frac{V_{out}}{(1-D)}\tilde{d} - \tilde{v}_o}{sL_{eq}} \Leftrightarrow$$

$$\Leftrightarrow \tilde{v}_o\left(\frac{1+sRC}{R} + \frac{1}{sL_{eq}}\right) = \frac{V_{out}}{(1-D)}\cdot\left(\frac{1}{sL_{eq}} - \frac{1}{R}\right)\tilde{d} \qquad \omega_{RHPZ} = \frac{R}{L_{eq}}$$

$$\Leftrightarrow \frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{out}}{(1-D)}\cdot\frac{\left(1 - s\frac{L_{eq}}{R}\right)}{s^2 L_{eq} C + s\frac{L_{eq}}{R} + 1} = \frac{1 - s\cdot{}^1/\omega_{RHPZ}}{\frac{s^2}{\omega_0^2} + s\frac{1}{\omega_0}\frac{1}{Q} + 1}$$

$$\frac{V_{out}\cdot(1-D)V_C A}{(1-D)V_C^2 A} = \frac{V_{out}}{V_C} = A$$

$$V_{IN} = (1-D)V_C A \Rightarrow V_{IN} + \tilde{v}_{im} = (1-D-\tilde{d})(V_C + \tilde{v}_C)A$$

$$V_{IN} = V_C A - D V_C A$$

$$\tilde{v}_{im} = \tilde{v}_C A - D\tilde{v}_C A - \tilde{d} V_C A$$

$$\tilde{d} = \frac{\tilde{v}_C A(1-D) - \tilde{v}_{im}}{V_C A}$$

$$\tilde{d} = -\frac{1}{V_C A}\tilde{v}_{im} + \frac{V_{IN}}{A\cdot V_C^2}\tilde{v}_C$$

$$G_{oC} = \frac{\tilde{v}_o}{\tilde{v}_C} = \frac{\tilde{v}_o}{\tilde{d}}\cdot\frac{\tilde{d}}{\tilde{v}_C} = \frac{V_{out}}{1-D}\cdot\frac{1 - s\cdot{}^1/\omega_{RHPZ}}{\frac{s^2}{\omega_0^2} + s\frac{1}{\omega_0}\frac{1}{Q} + 1}\cdot\frac{V_{IN}}{A\cdot V_C^2} =$$

$$A\cdot\frac{1 - s\cdot{}^1/\omega_{RHPZ}}{\frac{s^2}{\omega_0^2} + s\frac{1}{\omega_0}\frac{1}{Q} + 1} \quad //$$

$$G_{oL} = G_{od}\cdot\frac{\tilde{d}}{\tilde{v}_{im}} + G_{oL} = \frac{s}{\omega_z}G_{oL} = \frac{s}{\omega_z}\cdot\frac{1}{1-D}\cdot\frac{1}{1 + \frac{1}{Q}\frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}}$$