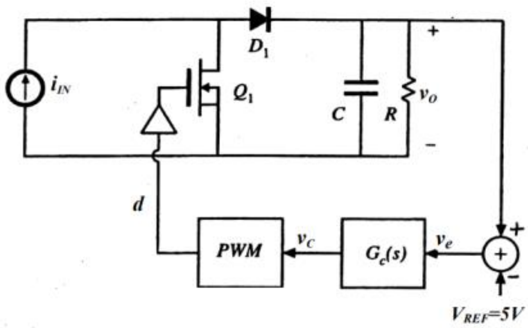
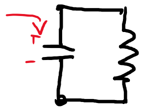
1) The current-fed converter shown in Fig. 1 is operating at f_{sw}=1 MHz (assume ideal MOSFETs and diodes).
a) Derive the DC transfer function V_O/I_{IN}.
Consider the following specifications: I_{IN} = 5A, V_O=5 V, P_O = 10 W.
b) Select the output capacitor C such that the peak-to-peak voltage ripple is less than 1% of the nominal voltage.



a) ON

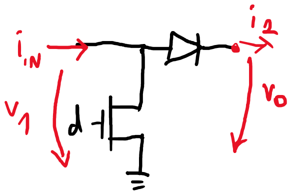


$$-I_0 \cdot D + (I_{IN} - I_0)(1-D) = 0$$
$$-\cancel{I_0 D} + I_N(1-D) - I_0 + \cancel{I_0 D} = 0$$
$$I_0 = \frac{V_0}{R} \qquad I_N(1-D) - \frac{V_0}{R} = 0$$
$$\frac{V_0}{I_N} = R(1-D)$$

b) $i = C\frac{\Delta V}{\Delta t}$

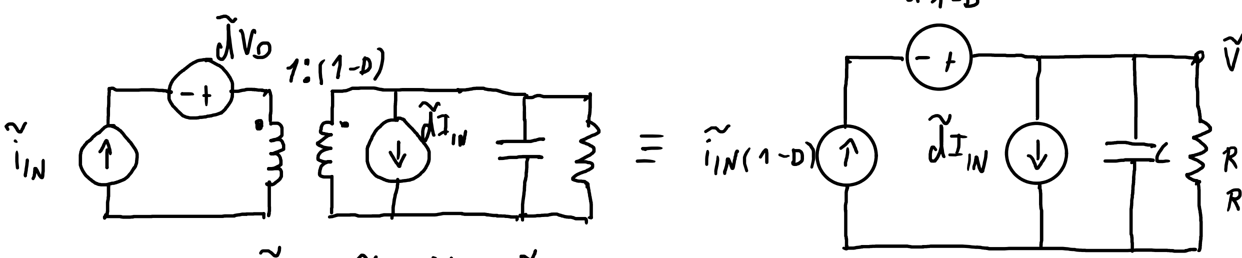
$$\frac{i\Delta t}{C} \le 0.01\, V_0 \Rightarrow C \ge \frac{I_0 D T_s}{0.01 \cdot V_0} \Leftrightarrow C \ge \frac{2 \cdot 0.6 \cdot 1/1M}{0.01 \cdot 5} \Leftrightarrow C \ge 24\, nF$$

c) Derive the transfer functions $G_{oc}(s) = \frac{\tilde{v}_o}{\tilde{v}_c}$ and $G_{vi}(s) = \frac{\tilde{v}_o}{\tilde{i}_{IN}}$, assuming $G_{PWM}(s)$=0.5 V^{-1} (**hint**: perturb and linearize the average diode current $\langle i_d \rangle_{T_s}$).



$$i_2 = (1-d)\, i_{IN}$$
$$v_1 = (1-d)\, v_0$$

$$I_2 + \tilde{i}_2 = (1 - D - \tilde{d})(I_{IN} + \tilde{i}_{IN})$$
$$V_1 + \tilde{v}_1 = (1 - D - \tilde{d})(V_0 + \tilde{v}_0)$$
$$I_2 = I_{IN}(1-D) \qquad \tilde{i}_2 = (1-D)\tilde{i}_{IN} - \tilde{d} I_{IN}$$
$$V_1 = V_0(1-D) \qquad \tilde{v}_1 = (1-D)\tilde{v}_0 - \tilde{d} V_0$$



$$R//C = \left(\frac{1}{R} + j\omega C\right)^{-1} = \frac{R}{1+sRC}$$

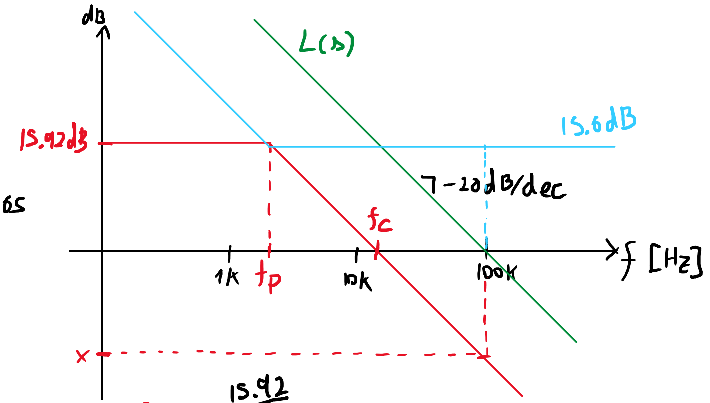
$$G_{oc}(s) = \frac{\tilde{v}_0}{\tilde{v}_c} = \frac{\tilde{v}_0}{\tilde{d}} \cdot \frac{\tilde{d}}{\tilde{v}_c} \Rightarrow \frac{\tilde{v}_0}{\tilde{d}}\Big|_{\tilde{i}_{IN}=0} = -\frac{R \cdot I_{IN}}{1+sRC} \Rightarrow G_{oc}(s) = -\frac{1}{2} \cdot \frac{R \cdot I_{IN}}{1+sRC}$$

$$G_{PWM}(s) = \frac{\tilde{d}}{\tilde{v}_c} = 0.5\, V^{-1}$$

$$G_{vi}(s) = \frac{\tilde{v}_0}{\tilde{i}_{IN}}\Big|_{\tilde{d}=0} = \frac{(1-D)R}{1+sRC}$$

d) Determine the compensator transfer function G_c (s) such that the loop gain has a constant slope of -20 dB / dec, crossing the 0dB axis at f_c = 100 kHz.

$100k \cdot 10^{\frac{x-0}{20}} = 2,65$



$G_{oc}(s) = -\frac{1}{2} \cdot \frac{R \cdot I_{IN}}{1 + sRC}$

$|G_{oc}(s)| = \frac{1}{2} R I_{IN} = 15.92\,dB$

$f_p = \frac{1}{2\pi RC} = 2,65\,kHz = f_z$

$L(s) = G_{oc}(s) \cdot G_s(s)$

$G_s(s) = K_c \cdot \frac{1 + s/\omega_z}{s}$ $\quad K_c \approx 100k$

$f_c = 10^{\frac{15.92}{20}} \cdot f_p = 16,58\,kHz$

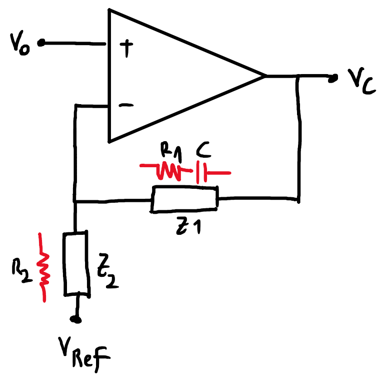
$100k = 10^{\frac{15.92 - x}{20}} \cdot f_p \Leftrightarrow 20 \log\left(\frac{100k}{2,65k}\right) = 15.92 - x \Leftrightarrow x = -15.62$

$L(s) = K_c \cdot \frac{1 + s/\omega_z}{s} \cdot -\frac{1}{2} \cdot \frac{R \cdot I_{IN}}{1 + sRC} = -\frac{1}{2} \cdot \frac{K_c \cdot R \cdot I_{IN}}{s} = -\frac{6.28 \times 10^5}{s}$

e) Design the error amplifier.

f) Sketch the asymptotic plot of the closed-loop output impedance.

e)



$\frac{\tilde{v}_c}{\tilde{v}_o} = 1 + \frac{Z_1}{Z_2} = \frac{Z_2 + Z_1}{Z_2} \sim K_c \cdot \frac{1 + s/\omega_z}{s}$

$Z_1 = R_1 + \frac{1}{sC} \Rightarrow \frac{R_2 + R_1 + \frac{1}{sC}}{R_2} = \frac{sC(R_2 + R_1) + 1}{sCR_2}$

$Z_2 = R_2$

$\frac{1}{K_c} = CR_2$ $\quad C = 10\,mF \quad R_2 = 1\,k\Omega \quad R_1 = 5\,k\Omega$

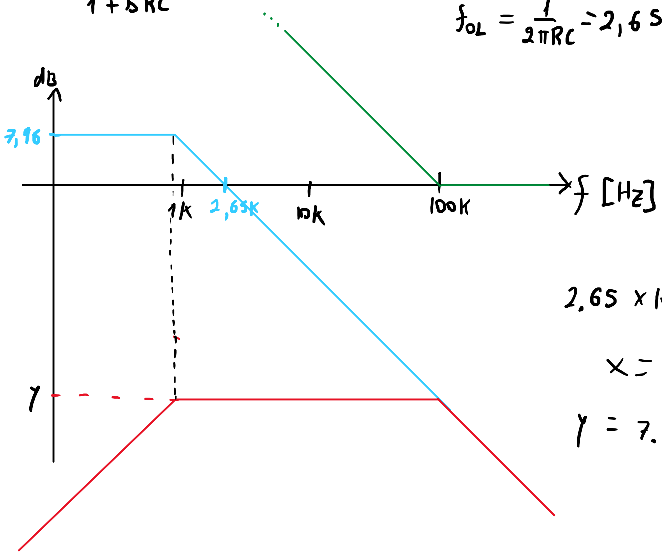
$\frac{1}{\omega_z} = C(R_2 + R_1)$

f) $Z_o^{OL} = \frac{R}{1 + sRC}$ $\quad C = 24\,\mu F$, $R = 2.5\,\Omega$

$Z_o^{OL}(0) = R = 7.96\,dB$

$f_{oL} = \frac{1}{2\pi RC} = 2,65\,kHz$

$1 + L(s) = 6.28 \times 10^5 \cdot \frac{\frac{s}{6,28 \times 10^5} - 1}{s}$



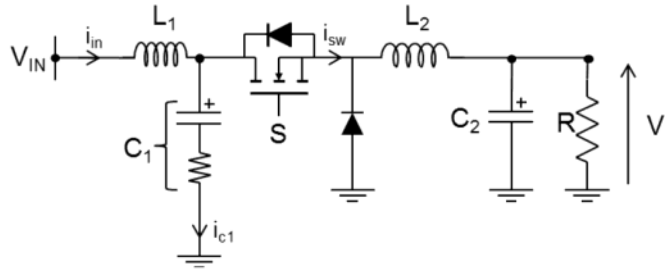
$Z_o^{CL} = \frac{Z_o^{OL}}{1 + L(s)}$

$2,65 \times 10^{\frac{x}{20}} = 100$

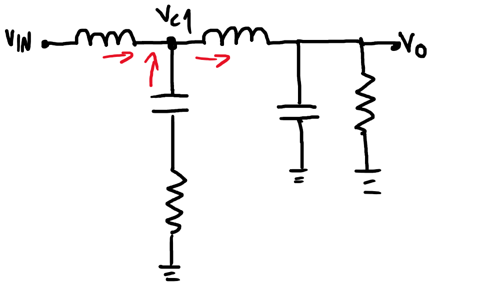
$x = 20 \log\left(\frac{100}{2,65}\right) = 31.5$

$Y = 7.96 - x = -23.57\,dB$

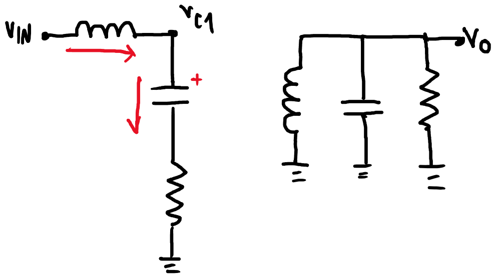
2) The buck converter shown in Fig. 1 operates in CCM at f_{SW} = 500kHz. The current ripple in L_2 is negligible. Assuming V_{IN} = 12V, V_O = 5 V, P_O = 15W, C_1 = 270µF (ESR = 22 mΩ, V_{rated}=25V, $I_{rmsrated}$=1520 mA_{rms}):
a) draw a plot of i_{C1} as a function of time;
b) calculate the voltage ripple ΔV_{C1} ;
c) calculate the average power dissipated by C_1;
d) select the inductance L_1 such that the peak-to-peakcurrent ripple ΔI_{L1} is less than 6 mA.



a) ON



OFF

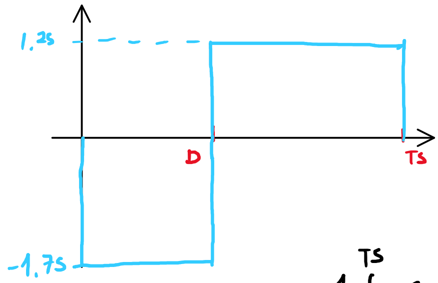


$(V_{IN} - V_{C_1}) D + (V_{IN} - V_{C1})(1-D) = 0 \Rightarrow V_{IN} = V_{C1}$

$(V_{C1} - V_0) D - V_0 (1-D) = 0 \Leftrightarrow V_{IN} D - V_0 = 0 \Rightarrow$

$\frac{V_o}{V_{IN}} = D = \frac{5}{12} = 0.4166$ ON $I_{L_1} + I_{c} = I_o \Rightarrow I_{c_1} = 1{,}75A$

$I_{L_1} = I_{IN} = D I_0 = 1.25A$ OFF



b) $\Delta V_{C_1} = \Delta V_C + \Delta V_{ESR} = 71.4mV$

$\Delta V_{ESR} = ESR \cdot \Delta I = 22 \cdot 10^{-3} \cdot (1.25 + 1.75) = 66\ mV$

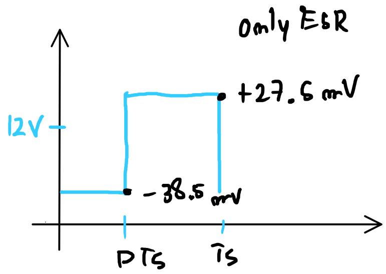
$\Delta V_C = \frac{I_0 D(1-D) T_s}{C} = 5.4mV$

c) $I_{c_1 rms}^2 = \frac{1}{T_S}\int_0^{T_S} I_{c_1}^2 = \frac{1}{T_S} \cdot \left[\int_0^{DT_S} (I_o(1-D))^2 + \int_{DT_S}^{T_S} (I_o \cdot D)^2 \right] =$

$= I_0^2 (1-D)^2 D + I_0^2 D^2 (1-D) = I_0^2 D(1-D)$

$P_{C_1} = (I_{c_1 rms})^2 \cdot ESR = I_o^2 D(1-D) ESR = 48.125\ mW$

d)



$V_{L1} = V_{IN} - V_{C1}$

$V_{L1}|_{ON} = -38.5mV - \left(\frac{\Delta V_C}{2}\right) \simeq -41.2mV$

$V_{L1}|_{OFF} = 27.5mV + \left(\frac{\Delta V_C}{2}\right) \simeq 30.2mV$

$\Delta i_{L_1} = \frac{V_{L1}|_{ON} \cdot DT_S}{L_1}$

$L_1 \geq \frac{V_{L1}|_{ON} DT_S}{\Delta i_{L_1}} = 5.347 \mu H$

$L_1 \geq 5.722 \mu H$