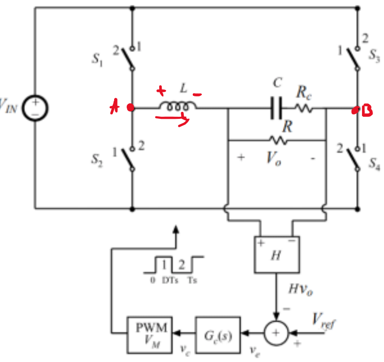
1) Consider the full-bridge converter shown in Fig. 1. V_{IN}= 48 V, f_{sw}= 500 kHz, C=470 μF, R_c=50 mΩ, L=50 μH, V_O= 5 V, P_O= 25 W, V_{ref}=1.2 V, V_M=2 V.
a) Derive the DC transfer function V_O/V_{IN}.
b) Calculate the peak-to-peak output voltage ripple.



a) $(V_{IN}-V_O)D + (0 - V_{IN} - V_O)(1-D) = 0 \iff$

$\iff V_{IN}D - V_{IN} + V_{IN}D - \cancel{V_O D} - V_O + \cancel{V_O D} = 0 \iff$

$\iff V_{IN}(2D-1) = V_O \iff \dfrac{V_O}{V_{IN}} = (2D-1)$

$D = 0.552 \quad R = 1$

$G_{PWM} = \dfrac{1}{V_M}$!

b) $\Delta V_O \cong \Delta V_C + \Delta V_{R_C}$

$\Delta I_L = \dfrac{(V_{IN}-V_O)}{L} \cdot D \cdot T_S = 949\,mA$

$\Delta V_{RC} = \Delta I_L \cdot R_C = 47,5\,mV$

$\Delta V_C = \dfrac{\Delta Q}{C} = \dfrac{\Delta I_L \cdot T_S}{8C} = 0,5\,mV \Rightarrow \Delta V_O = 48\,mV$

c) Derive the transfer function $G_{oc}(s) = \dfrac{\tilde{v}_O}{\tilde{v}_C}$, (**hint**: perturb and linearize the bridge voltage $\langle v_{AB} \rangle_{T_S}$).

Sketch the Bode plot of $G_{oc}(s)$.

$\langle v_{AB} \rangle = d \cdot \langle v_{IN} \rangle - (1-d)\langle v_{IN} \rangle = (2d-1)\langle v_{IN} \rangle$

$\langle v_{AB} \rangle = V_{AB} + \tilde{v}_{AB}$

$\langle v_{IN} \rangle = V_{IN} + \tilde{v}_{IN}$

$d = D + \tilde{d}$

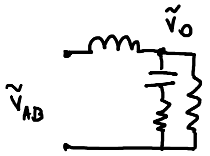
$\Rightarrow \begin{cases} V_{AB} = (2D-1)V_{IN} \\ \tilde{v}_{AB} = (2D-1)\tilde{v}_{IN} + 2V_{IN}\tilde{d} \end{cases}$

$\tilde{d} = \dfrac{\tilde{v}_C}{V_M} = \dfrac{\tilde{v}_C}{2}$

$Z_1 = sL$

$Z_2 = \dfrac{(sCR_c+1)R}{sCR + sCR_c + 1}$

$\tilde{v}_O = \tilde{v}_{AB} \cdot \dfrac{Z_2}{Z_2 + Z_1}$



$\left(\dfrac{Z_2}{Z_2+Z_1}\right)^{-1} = (Z_2+Z_1)\cdot\dfrac{1}{Z_2} = 1 + sL \cdot \dfrac{sCR + sCR_c + 1}{(sCR_c+1)R}$

$R >> R_c$

$G_{oc}(s) = \dfrac{\tilde{v}_O}{\tilde{v}_C}\Big|_{\tilde{v}_{IN}=0} \iff \tilde{v}_O\left(1 + sL \cdot \dfrac{sCR + \cancel{sCR_c} + 1}{(sCR_c+1)R}\right) = V_{IN}\tilde{v}_C \iff \dfrac{\tilde{v}_O}{\tilde{v}_C} = V_{IN} \cdot \dfrac{sCR_c + 1}{s^2LC(1+\frac{R_c}{R}) + s(\frac{L}{R} + CR_c) + 1}$

$\omega_{ESR} = \dfrac{1}{CR_c} = 42.5\,K\,rad/s \Rightarrow f_{ESR} = 6.77\,kHz$

$G_{oc}(s) = V_{IN} \cdot \dfrac{(1 + \frac{s}{\omega_z})}{\frac{s^2}{\omega_0^2} + s\frac{2\zeta}{\omega_0} + 1}$

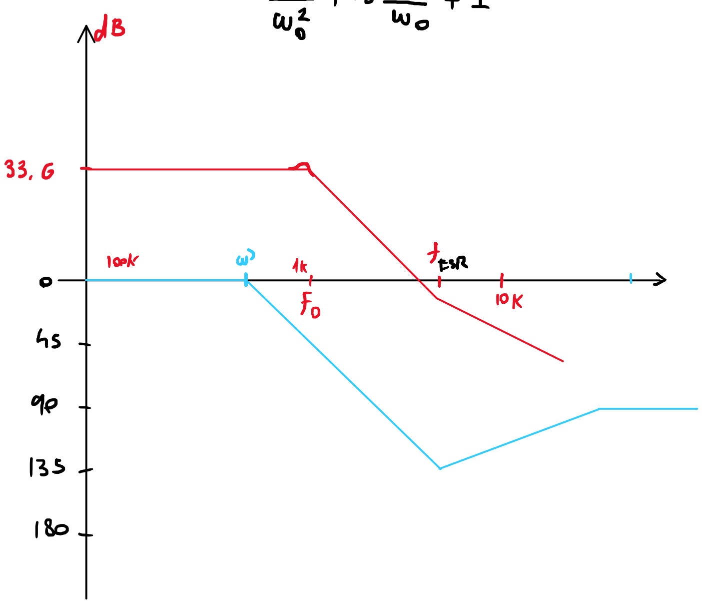
$\omega_0 = \dfrac{1}{\sqrt{LC(1+\frac{R_c}{R})}} \Rightarrow f_0 = 1.01\,kHz$

With $R_c << R \Rightarrow f_0 = 1.04\,kHz$

$\zeta = (\frac{L}{R} + CR_c)\dfrac{\omega_0}{2} = 0.20$

$\omega' = \omega_0 \cdot 10^{-\zeta} = 630\,Hz$

$\omega'' = \omega_0 \cdot 10^{\zeta} = 1.6\,kHz$



d) Derive H such that V_O=5 V.

e) Using the k-factor design method, determine the type-II compensator transfer function, $G_c(s)$, such that the loop transfer function crosses the 0dB axis at f_c = 50 kHz with a phase margin φ_m=60°.

(note: $k = \sqrt{\frac{\omega_p}{\omega_z}} = \tan\left(\frac{\varphi_{boost}}{2} + \frac{\pi}{4}\right)$).

d) $HV_o = V_{Ref} \Leftrightarrow H = \frac{1.2}{5} = 0.240$

$\angle G_{oc}(j\omega_c) = -2\arctan\left(\frac{f_c}{f_0}\right) + \arctan\left(\frac{f_c}{f_{ESR}}\right) = -95.4^\circ$

e) $f_c = 50kHz$

$\varphi_m = 60^\circ$

$|G_{oc}(f_c)| = V_{IN} \cdot \left(\frac{f_0}{f_{ESR}}\right)^2 \cdot \frac{f_{ESR}}{f_c} = 0.153 = -16.3\,dB$

$\varphi_b = \varphi_m - \angle G_{oc}(j\omega_c) - 90^\circ = 65^\circ$

$K = \sqrt{\frac{\omega_p}{\omega_z}} = \tan\left(\frac{\varphi_b}{2} + \frac{\pi}{4}\right) = 4.51$

$\omega_z = \frac{\omega_c}{K} = 2\pi \cdot \frac{50}{4.51}k = 69.7\ krad/s$

$\omega_p = \omega_c K = 2\pi \cdot 50 \cdot 4.51k = 1.41\ Mrad/s$

$|G_c(j\omega_c)| = \frac{1}{|G_{oc}(j\omega_c)|\,H} = 27.23$

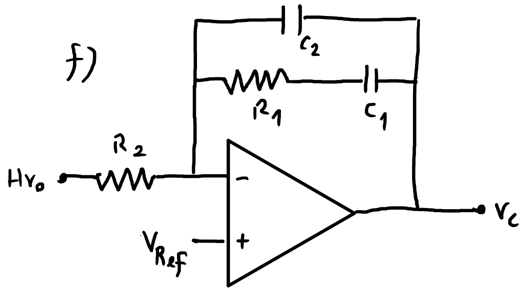
$G_c(s) = \frac{K_c}{s} \cdot \frac{1 + s/\omega_z}{1 + s/\omega_p}$

$K_c = \omega_c \cdot \frac{\sqrt{1 + 1/k^2}}{\sqrt{1 + k^2}} \cdot |G_c(j\omega_c)| = 1.9\ Mrad/s$

f) Design the error amplifier/compensator.

g) The input voltage is varied between 36 and 72 V. What are the issues with system stability?

f)



$G_c(s) = -\frac{1}{R_2} \cdot \frac{sC_1R_1 + 1}{sC_2(sC_1R_1 + 1) + sC_1} =$

$= -\frac{1}{sR_2C_1} \cdot \frac{1 + sC_1R_1}{sC_2R_1 + 1 + \underbrace{\frac{C_2}{C_1}}_{\approx 0 \text{ if } C_1 \gg C_2}}$

$R_1C_1 = 1/\omega_z$ | $R_1 = 27k\Omega$

$R_1C_2 = 1/\omega_p$ | $C_2 = 26mF$

$R_2C_1 = 1/K_c$ | $C_1 = 520mF$

$R_2 = 1k\Omega$

g) VIN ↗ => $|G_{oc}(s)|$ ↗

Garantee: worst => ?

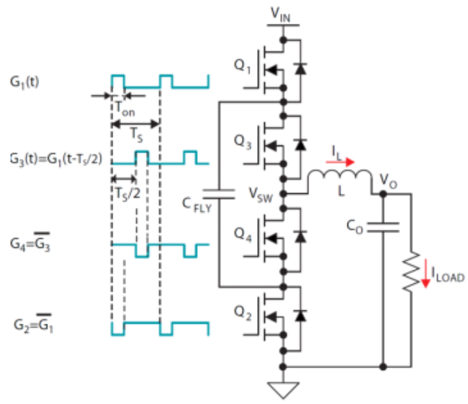
$f_c < \frac{1}{5} f_{sw}$ To be safe a factor of 10 is sometimes used

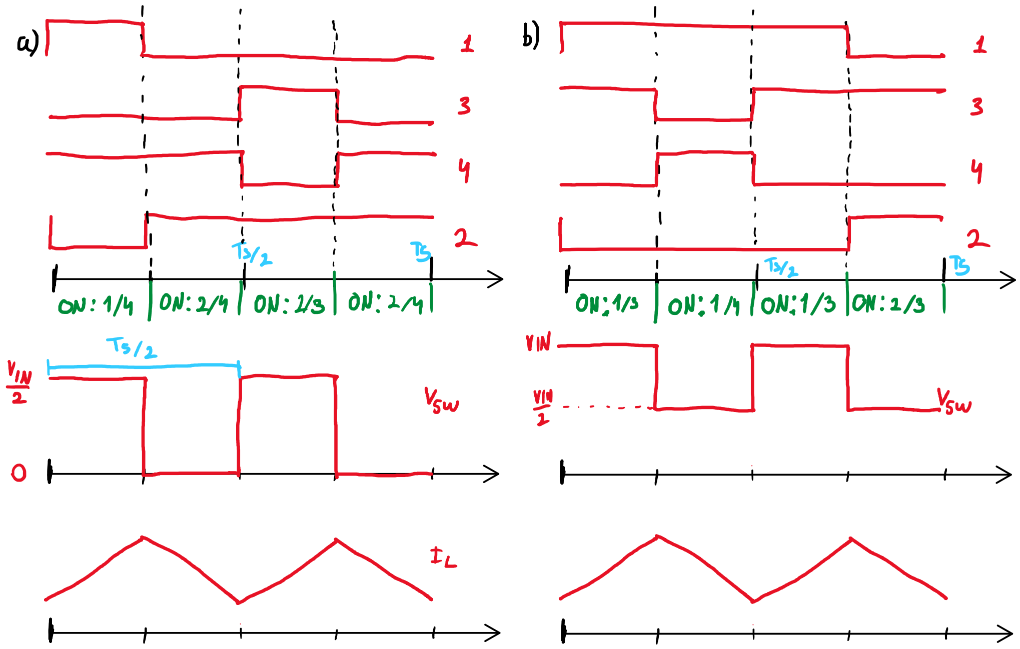
stability

After $\frac{1}{3}$ of f_{sw} the model goes to shit

for both sides decrease or increase we lose phase margin!

2) The three-level buck converter shown in Fig. 2 operates in CCM at $f_{sw} = 1/T_s = 750$ kHz. The switch gate-driving waveforms are illustrated in Fig. 2 as well ($D=T_{on}/T_S$). Assume the converter is lossless.
a) Sketch the plot of V_{SW} and I_L as a function of time, assuming $V_{CFLY}=V_{IN}/2$ and $D<0.5$.
b) Sketch the plot of V_{SW} and I_L as a function of time, assuming $V_{CFLY}=V_{IN}/2$ and $D>0.5$.
c) Derive the DC voltage transfer function V_O/V_{IN} as a function of D.
d) Derive an expression for the peak-to-peak inductor current ripple, Δi_{L_3L} as a function of D.
e) Sketch the plot of Δi_{L_3L} as a function of D, along with the plot of Δi_{L_2L} for a standard buck converter.
What is the advantage of the three-level buck compared with the standard (two-level) buck converter?





c) $(V_{sw}-V_o)D + (V_{sw}-V_o)(\frac{1}{2}-D) = 0$

$D<0{,}5$:

$$\left(\frac{V_{IN}}{2}-V_o\right)D + (0-V_o)\left(\frac{1}{2}-D\right)=0$$

$$V_{IN}D = V_o \iff \frac{V_o}{V_{IN}} = D$$

$D>0{,}5$:

$$(V_{IN}-V_o)\left(D-\frac{1}{2}\right) + \left(\frac{V_{IN}}{2}-V_o\right)(1-D)=0$$

$$V_{IN}\left(\cancel{D}-\cancel{\tfrac{1}{2}}+\cancel{\tfrac{1}{2}}-\tfrac{D}{2}\right) - V_o\left(\cancel{D}-\tfrac{1}{2}+\cancel{1}-\cancel{D}\right)=0$$

$$\frac{V_o}{V_{IN}} = D$$

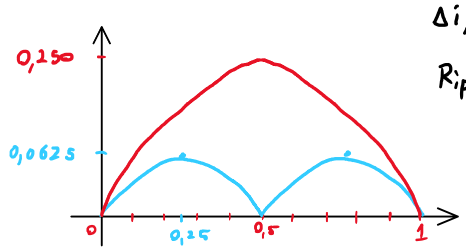
d) $V = L\frac{di}{dt} \iff \Delta i = \frac{V\Delta t}{L} = \frac{(\frac{V_{IN}}{2}-V_o)DT_s}{L} = \frac{V_{IN}(\frac{1}{2}-D)\cdot D}{L f_{sw}}$ $\quad D<0.5$

$D>0.5$ $\quad \frac{(\frac{V_{IN}}{2}-V_o)(1-D)}{L f_{sw}} = \frac{V_{IN}(\frac{1}{2}-D)(1-D)}{L f_{sw}}$

e) $\Delta i_{L2} = \frac{(V_{IN}-V_o)D}{L f_{sw}} = \frac{V_{IN}(1-D)D}{L f_{sw}}$

$\Delta i_2 \text{ norm} = (1-D)D$

$$\Delta i_3 \text{ norm} = \begin{cases} (\frac{1}{2}-D)D \\ (\frac{1}{2}-D)(1-D) \end{cases}$$



$\Delta i_{L_2}\text{Max} = 4\cdot\Delta i_{L_3}\text{Max}$

Ripple is 4 times smaller

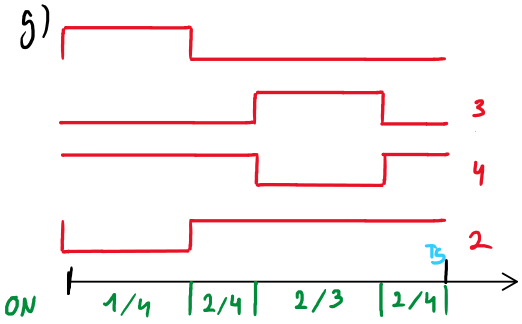
Assuming $V_{IN} = 12V$, $V_O = 3.8V$, $P_O = 7.6$ W:
f) Select the inductance L such that the peak-to-peak current ripple is less than 30% of the average current.
g) Sketch a plot of the current flowing through C_{FLY} as a function of time.
h) Select C_{FLY} to ensure that the peak-to-peak voltage ripple is less than 10% of the nominal voltage.
i) Calculate the peak-to-peak output voltage ripple ($C_O = 22$ μF, ESR=5mΩ).

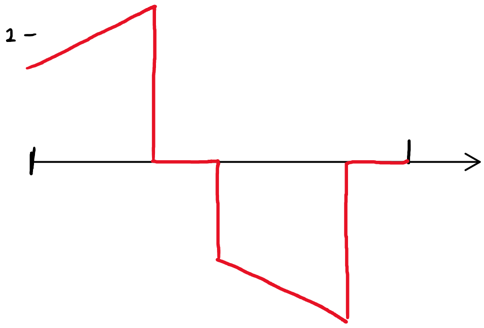
f) $I_o = 2$ A $\Rightarrow \Delta I_L = 0{,}6$ A

$D = 0.317$

$$\Delta I_L < \frac{V_{IN}(\frac{1}{2}-D)\cdot D}{L f_{sw}} \iff L < \frac{V_{IN}(\frac{1}{2}-D)D}{\Delta I_L f_{sw}} \iff L < 1.55\,\mu H$$

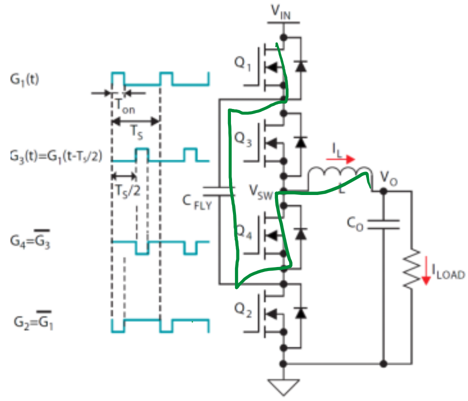
g)





h)

Neglecting current ripple



$$i = C\frac{\Delta V}{\Delta t} \iff \Delta V = \frac{i\Delta t}{C}$$

$$0.1\frac{V_{IN}}{2} < \frac{I_o D T_s}{C} \Rightarrow C < \frac{2 I_o D T_s}{0.1 V_{IN}}$$

$$C < 1.4\,\mu F$$

i)

$$\Delta V = \frac{1}{2}\cdot\frac{\Delta T}{4}\cdot\frac{\Delta I_o}{2}\cdot C + ESR\cdot\Delta I_o =$$

$\Delta I_L = 0,6\,A$

$$= \frac{\Delta I_o}{16 C f_{sw}} + ESR\,\Delta I_o \Rightarrow \Delta V = 2.27\,mV + 3\,mV$$