**1.)** $E(z,t) = E_0 \cos(\omega t - kz + \theta)\,\hat{y}$ $\qquad \hat{y} - (0,1,0)$

a) Propaga-se segundo $z$ → Varia em $z$

Polarização linear segundo Y → é um vetor orientado em $\hat{y}$

Monocromática → só tem uma frequência

Onda plana → só varia em $z$ e $t$

b) $E_0$ - amplitude máxima do campo elétrico

$\omega$ - frequência angular

$k$ - constante de propagação

$\theta$ - fase na origem

c) Propaga-se em $z$

Está orientado segundo Y

d) $E(z) = E_0 e^{j(\theta - kz)}\,\hat{y}$ → (tal como AC in)

$$E = E_0 e^{j(\theta - kz)}$$

$$\frac{\partial E}{\partial z} = -jkE_0 e^{j(\theta - kz)}$$

$$\frac{\partial E^2}{\partial^2 z} = -k^2 E_0 e^{j(\theta - kz)} = -k^2 E$$

e)

$$\begin{cases} \nabla \times H = j\omega\varepsilon E \\ \nabla \times E = -j\omega\mu H \\ \nabla \cdot D = 0 \\ \nabla \cdot B = 0 \end{cases} \quad e \quad \nabla \times \nabla \times E = \nabla(\underbrace{\nabla \cdot E}_{0}) - \nabla^2 E = -\nabla^2 E$$

$$\to \begin{cases} \nabla \times H = j\omega\varepsilon E \\ -\dfrac{\nabla \times E}{j\omega\mu} = H \end{cases} \Leftrightarrow \left| \nabla \times \nabla \times E = \omega^2 \mu\varepsilon E \right. \Leftrightarrow \nabla^2 E + \mu\varepsilon\omega^2 E = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{\partial^2 E}{\partial z^2} + \omega^2 \mu\varepsilon E = 0 \Leftrightarrow -k^2 E + \omega^2 \mu\varepsilon E = 0 \Leftrightarrow E(\omega^2 \mu\varepsilon - k^2) = 0$$

$$\omega^2 \mu\varepsilon - k^2 = 0$$

$$\therefore \quad k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v}$$

f) $\omega t - kz + \theta = \alpha \Leftrightarrow z = \dfrac{\omega}{k}t + \dfrac{\theta - \alpha}{k}$

$v_f = \dfrac{dz}{dt} = \dfrac{\omega}{k} \Rightarrow v_f = v$ (pela alínea anterior)

$$\frac{\sqrt{\mu\varepsilon}}{\mu} = \sqrt{\frac{\varepsilon}{\mu}} = \frac{1}{Z_0}$$

g) $\nabla \times E = -j\omega\mu H$

$$H = \frac{\nabla \times E}{-j\omega\mu} = -\frac{1}{j\omega\mu}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ \underset{0}{E_x} & E_y & \underset{0}{E_z} \end{vmatrix} = -\frac{\partial E_y}{\partial z}\hat{x} + \overbrace{\frac{\partial E_y}{\partial x}}^{0}\hat{z} = \frac{1}{j\omega\mu}\cdot(-j)kE_0 e^{j(\theta - kz)} \Rightarrow H = -E_0\frac{\sqrt{\mu\varepsilon}}{\mu}e^{j(\theta - kz)} = -\frac{E_0}{Z_0}e^{j(\theta - kz)}\,\hat{x}$$

$$H = -\frac{E_0}{Z_0}e^{j(\theta - kz)}\,\hat{x} \quad e \quad H(z,t) = -\frac{E_0}{Z_0}\cos(\omega t - kz + \theta)$$

h) $\omega = 2\pi \cdot 10^8$

$k_0 = \dfrac{\omega}{c_0} = \dfrac{2\pi}{3}$

$\lambda_0 = \dfrac{c_0}{f} = 3$

**2.)** I a) $\lambda_0 = \dfrac{c_0}{f} \to [\lambda_0] = \dfrac{3 \times 10^8}{[535;1605] \times 10^3} = [0{,}561\ \text{Km}\ ;\ 0{,}187\ \text{Km}]$

II b) $\lambda_0 = \dfrac{c_0}{f} \to [\lambda_0] \Rightarrow [3{,}41\ \text{m}\ ;\ 2{,}78\ \text{m}]$

III c) $\lambda_0 = 0{,}025\ \text{m} = 2{,}5\ \text{cm}$

IV d) $f = \dfrac{c_0}{\lambda_0} = 200 \times 10^{12}\ \text{Hz} = 200\ \text{THz}$

V e) $\lambda = [380\ \text{nm}, 780\ \text{nm}] \Rightarrow f = [789{,}475\ \text{THz}\ ,\ 384{,}615\ \text{THz}]$

f)

Vermelho : $\lambda = 700$ nm → $f = 428{,}571$ THz

laranja : $\lambda = 610$ nm → $f = 491{,}803$ THz

amarelo : $\lambda = 590$ nm → $f = 508{,}475$ THz

verde : $\lambda = 530$ nm → $f = 566{,}038$ THz

azul : $\lambda = 470$ nm → $f = 638{,}298$ THz

violeta : $\lambda = 420$ nm → $f = 714{,}286$ THz

**3.)** I

$E(z,t) = E_0 \cos\left[10^8\pi\left(t - \frac{z}{c}\right) + \theta\right]\hat{x}$

a) Propaga-se em $z$ $\qquad f = \dfrac{10^8\pi}{2\pi} = 5 \times 10^7 = 50\ \text{MHz}$

Polarizado segundo x $\qquad k = \dfrac{10^8\pi}{c} = \dfrac{\pi}{3}\ \text{Rad/m}$ $\qquad k = \omega\sqrt{\mu\varepsilon}$

b) $E(z) = E_0 e^{j(\theta - kz)}\,\hat{x}$

$\dfrac{\partial E}{\partial z} = -jkE_0 e^{j(\theta - kz)}\,\hat{x}$

$H = \dfrac{\nabla \times E}{-j\omega\mu} = -\dfrac{1}{j\omega\mu}\cdot\dfrac{\partial E_x}{\partial z}\hat{y} = \dfrac{E_0}{Z_0}e^{j(\theta - kz)}\,\hat{y}$

$H = \dfrac{E_0}{Z_0}e^{j(\theta - kz)}\,\hat{y}$

c) $Z_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 120\pi\ \Omega$ $\qquad H(z,t) = \dfrac{E_0}{Z_0}\cos(\omega t - kz + \theta)\,\hat{y}$

$\theta_E - \theta_H = \theta - \theta = 0$

d) $\omega t - kz + \theta = \alpha \Leftrightarrow z = \dfrac{\omega}{k}t + \dfrac{\theta - \alpha}{k}$

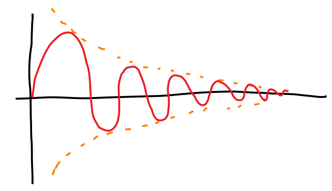
$v_f = \dfrac{dz}{dt} = \dfrac{\omega}{k} = c_0$

II e) $\varepsilon = \varepsilon_0 \mid \mu = \mu_0 \mid \sigma = ?$

$\gamma = \alpha + \beta j$

$E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z + \theta)\,\hat{x}$

f) $E(z) = E_0 e^{-\alpha z}\cos(-\beta z + \theta)$



g) $v_f = \dfrac{\omega}{\beta} = 50{,}27 \times 10^3$ m/s

h) $\alpha = \beta \Rightarrow$ Bom condutor $\Rightarrow \sigma >> \omega\varepsilon$

$$\gamma = \alpha + \beta j = j\omega\sqrt{\mu\varepsilon}\cdot\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}} \approx (1+j)\sqrt{\pi f \mu\sigma}$$

$$Z = \sqrt{\frac{\mu}{\varepsilon^*}} = \sqrt{\frac{\mu}{\varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)}} \simeq \sqrt{\frac{\mu}{\frac{\sigma}{j\omega}}} = \sqrt{\frac{j\mu\omega}{\sigma}} =$$

$$= (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = \frac{\omega\mu}{2\beta}(1+j) = 44{,}66\, e^{j\frac{\pi}{4}}\ \text{m}\Omega$$

$$\theta_E - \theta_T = \measuredangle(z) = 45^\circ = \frac{\pi}{4}$$

**4.)** I $\quad \varepsilon = 4\varepsilon_0 \quad f = 100$ MHz

$\mu = \mu_0$

$\sigma = 0 \qquad t = 0;\ z = 0{,}125 \to$ Max

$c = \dfrac{c_0}{\sqrt{\mu_r\varepsilon_r}} = \dfrac{c_0}{2} = 1{,}5 \times 10^8 \qquad k = \dfrac{\omega}{c} = \dfrac{4\pi}{3}$ Rad/m

$\omega t - kz + \theta = 0$ é um Máx → $\theta = \dfrac{4\pi}{3}\cdot 0{,}125 = \dfrac{\pi}{6}$

$\vec{E}(z) = E_0 e^{-j\left(\frac{4\pi}{3}z - \frac{\pi}{6}\right)}\,\hat{x} \quad e \quad \vec{E}(z,t) = E_0\cos\left(2\pi \cdot 10^8 t - \dfrac{4\pi}{3}z + \dfrac{\pi}{6}\right)\hat{x}$

$H = \dfrac{\hat{z} \times E}{Z} = \dfrac{E_x}{Z} \to \vec{H}(z) = \dfrac{E_0}{Z}e^{-j\left(\frac{4\pi}{3}z - \frac{\pi}{6}\right)}\,\hat{y} \quad e \quad \vec{H}(z,t) = \dfrac{E_0}{Z}\cos\left(2\pi \cdot 10^8 t - \dfrac{4\pi}{3}z + \dfrac{\pi}{6}\right)\hat{y}$

b) $\omega t - kz + \theta = \pm 2\pi m$ é Max $\quad \dfrac{\omega t + \theta \pm 2\pi m}{k} = z = 1{,}625 \pm 1{,}5m$ [m] !

$m = 0, 1, 2$

II

c) $\varepsilon = 4\varepsilon_0$ $\qquad E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z + \theta)\,\hat{x}$

$\mu = \mu_0$ $\qquad \alpha = \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon}} = 10{,}472 \times 10^{-3}$ Np/m

$\tan(\theta_\delta) = \dfrac{\sigma}{\omega\varepsilon} = 0{,}005 << 1$! Perdas fracas $\qquad \beta = \dfrac{\omega}{c_0}\sqrt{\mu_r\varepsilon_r} = \dfrac{4\pi}{3}$ Rad/m

**.5)** $f = 100\,GHz$

ar  1) $\varepsilon = \varepsilon_0$ , $\mu = \mu_0$

Bom dielétrico  2) $\varepsilon = 2{,}25\,\varepsilon_0$ , $\mu = \mu_0$ , $\tan(\theta_p) = 0{,}002$

Bom condutor  3) $\varepsilon = \varepsilon_0$ , $\mu = \mu_0$ , $\sigma = 5{,}8\times10^7\ S/m$

a) 1) $\gamma = \alpha + j\beta = j\frac{2000\pi}{3}$ | $Z = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi$ | $V_f = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} = c = 3\times10^8$

$\alpha = 0$

$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2000\pi}{3}\ rad/m$

2) $\gamma = \alpha + j\beta = \pi + j1000\pi$

$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\tan(\theta_p)\cdot\omega\cdot\varepsilon}{2}\cdot\sqrt{\frac{\mu}{\varepsilon}} = \pi$ | $Z = \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right) = 80\pi(1 + 0.001j)$ | $V_f = \frac{\omega}{\beta} = 200\times10^6$

$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} = 1000\pi$

3) $\gamma = \alpha + j\beta = (4{,}785 + 4{,}785j)\times10^6$ | $Z = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = 8{,}25\times10^{-2}(1+j)$ | $V_f = \frac{\omega}{\beta} = 1.31\times10^5$

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = 4{,}785\times10^6$

**.6)** I $\varepsilon = \varepsilon_0$  $\sigma = 5{,}8\times10^7$

$\mu = \mu_0$

a) $\tan(\theta_p) = \frac{\sigma}{\omega\varepsilon}$ será quase sempre >> 1 logo é um bom condutor ($\sigma$ elevado também indica isso)

Logo: $Z = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$   $E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z + \theta_E)\,\hat{x}$

$\gamma = \alpha + \beta j$   $E_0 = |Z|H_0$  $\theta_E = \arg(Z) = \frac{\pi}{4}$

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

b) $f_1 = 16\,kHz \rightarrow \omega_1 = 2\pi\cdot16K\ rad$   $f_2 = 1.6\,GHz \rightarrow \omega_2 = 2\pi\cdot1.6G\ rad$

$Z = 3.3\times10^{-6}(1+j)$   $Z = 10.44\times10^{-2}(1+j)$

$\alpha = \beta = 1.914\times10^3$   $\alpha = \beta = 6.053\times10^5$

II a) $\tan(\theta_p) = \frac{\sigma}{\varepsilon\omega} \rightarrow$ $f_1 \rightarrow \tan(\theta_p) = 70.3125\times10^3 >> 1$ ! Bom condutor

$f_2 \rightarrow \tan(\theta_p) = 0.7 \approx 1$  Equações normais

Logo $Z = \sqrt{\frac{\mu}{\varepsilon^*}}$

$\gamma = \alpha + j\beta = j\omega\sqrt{\varepsilon^*\mu}$

$\varepsilon^* = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$

$f_1$: 16 kHz

$Z = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = 0.112\,(1+j)\ \Omega$

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = 0.562$

$f_2$: 1.6 GHz

$\varepsilon^* = 80\varepsilon_0\left(1 - j\frac{\sigma}{80\varepsilon_0\omega}\right)$

$Z = \sqrt{\frac{\mu}{\varepsilon^*}} = 36.35 + j11.50\ \Omega$

$\gamma = j\omega\sqrt{\varepsilon^*\mu}$

$\alpha = \mathrm{Re}\{\gamma\} = 99.96\ Np/m$

$\beta = \mathrm{Im}\{\gamma\} = 315.96\ rad/m$

**.7)** $f_0 = 100\,MHz$  a) $\tan(\theta_p) = \frac{\sigma}{\varepsilon\omega} = 1.5\times10^6 \rightarrow$ Bom condutor

$\varepsilon = 12\varepsilon_0$

$\mu = \mu_0$   $\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{2000\pi} \simeq 159.2\ \mu m$

$\sigma = 10^6$

$\delta$ é a distância para a qual o campo atenua 1 Neper (36.7?)

b) 30 dB $\rightarrow$ atenuar 0,0316

$0{,}0316 = e^{-\alpha z} \Rightarrow z = \frac{\ln(0{,}0316)}{-\alpha} = 549.7\ \mu m$

c) $\theta_E - \theta_H = \measuredangle Z = \frac{\pi}{4}$

$Z = \sqrt{\frac{\omega\mu}{2\sigma}}\underbrace{(1+j)}_{}\ \rightarrow \measuredangle Z = \frac{\pi}{4}$

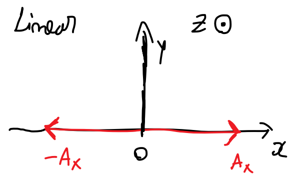
d) A desfasagem depende do argumento de $Z$ e $Z = \sqrt{\frac{\mu}{\varepsilon^*}}$ onde $\varepsilon^* = \varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right) \simeq \frac{\sigma}{j\omega}$ (como $\sigma >> \omega\varepsilon$ desprezamos 1) logo $Z = \sqrt{\frac{2\pi f\mu j}{\sigma}} = (1+j)\sqrt{\frac{\pi f\mu}{\sigma}}$

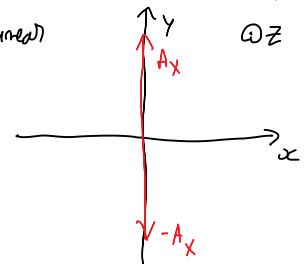
desfasagem não depende do $\sigma$ => É sempre $\frac{\pi}{4}$

$\hat{z} = \hat{n}$

**.4)** I a) $E = A_x\cos(\omega t - k_z z + \phi_x)\,\hat{x}$   b) $E = A_y\cos(\omega t - k_z z + \phi_y)\,\hat{y}$

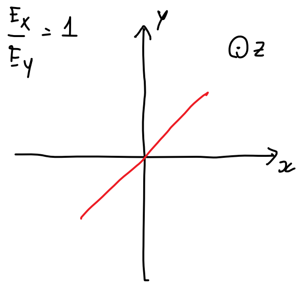
Linear   Linear

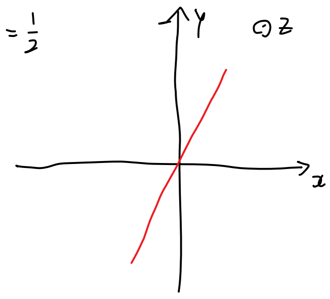



c) $E = A\cos(\omega t + \phi)\,\hat{x} + A\cos(\omega t + \phi)\,\hat{y}$   d) $E = A\cos(\omega t + \phi)\,\hat{x} + 2A\cos(\omega t + \phi)\,\hat{y}$

$\frac{E_x}{E_y} = 1$   $\frac{E_x}{E_y} = \frac{1}{2}$

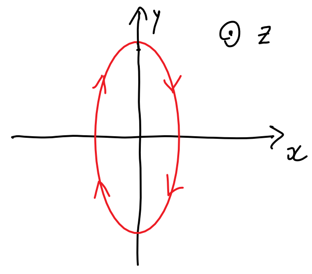



e) $E = A\cos(\omega t + \phi)\,\hat{x} + 2A\cos(\omega t + \phi + \frac{\pi}{2})\,\hat{y}$

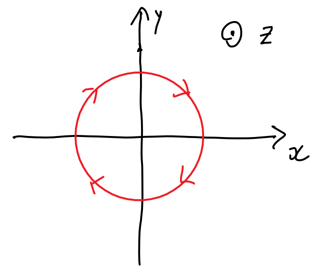
$A\cos(\omega t + \phi)\,\hat{x} - 2A\sin(\omega t + \phi + \frac{\pi}{2})\,\hat{y}$

$\left(\frac{E_x}{A}\right)^2 + \left(\frac{E_y}{2A}\right)^2 = 1$ | $\omega t + \phi = 0$: $E_x = A$, $E_y = 0$ | $\omega t + \phi = 0^+$: $E_x < A$, $E_y < 0$



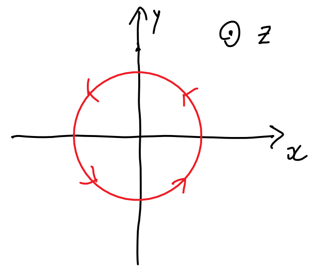

f) $E = A\cos(\omega t + \phi)\,\hat{x} + A\cos(\omega t + \phi + \frac{\pi}{2})\,\hat{y}$

$E_x^2 + E_y^2 = 1$ | $\omega t + \phi = 0$: $E_x = A$, $E_y = 0$ | $\omega t + \phi = 0^+$: $E_x < A$, $E_y < 0$




g) $E = A\cos(\omega t + \phi)\,\hat{x} + A\cos(\omega t + \phi - \frac{\pi}{2})\,\hat{y}$

$E_x^2 + E_y^2 = 1$ | $\omega t + \phi = 0$: $E_x = A$, $E_y = 0$ | $\omega t + \phi = 0^+$: $E_x < A$, $E_y > 0$

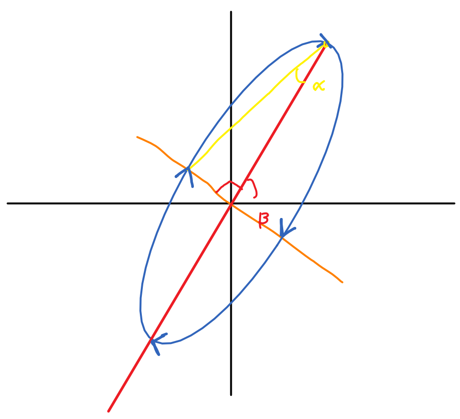



h) A seguinte resolução envolve conteúdos ainda não lecionados !!!

$E = A\cos(\omega t + \phi)\,\hat{x} + 2A\sin(\omega t + \phi + \frac{\pi}{4})\,\hat{y}$

$P = \frac{\bar{E}_y}{\bar{E}_x} = 2e^{j\frac{\pi}{4}} \rightarrow \tan(\gamma) = 2$, $\phi = \frac{\pi}{4}$ $\rightarrow \alpha = \frac{\arcsin\left(\sin(2\cdot\arctan(2))\cdot\sin(\phi)\right)}{2} = 17.2°$

$\beta = \frac{\arctan\left(\tan(2\arctan(2))\cdot\cos(\phi)\right)}{2} = 68.3°$




II) i) $Ae^{-jk_z z}(\hat{x} + j\hat{y}) + Ae^{-jk_z z}(\hat{x} - j\hat{y}) = 2Ae^{-jk_z z}\,\hat{x}$

ii) $Ae^{-jk_z z}(\hat{x} + j\hat{y}) - Ae^{-jk_z z}(\hat{x} - j\hat{y}) = 2Ae^{-j(k_z z - \frac{\pi}{2})}\,\hat{y}$

iii) $Ae^{-jk_z z}(\hat{x} + j\hat{y}) = Ae^{-jk_z z}\,\hat{x} + Ae^{-j(k_z z - \frac{\pi}{2})}\,\hat{y}$

iv) $Ae^{-jk_z z}(\hat{x} - j\hat{y}) = Ae^{-jk_z z}\,\hat{x} + Ae^{-j(k_z z + \frac{\pi}{2})}\,\hat{y}$

8) I a) $\vec{E}(z) = E_0 e^{-\alpha z} e^{-j\beta z}\,\hat{x}$

b) $\vec{H} = \hat{m} \times \frac{\vec{E}}{Z} = \frac{E_0}{Z} e^{-\alpha z} e^{-j\beta z}\,\hat{y}$

$\hat{m} = \hat{z}$ $\qquad Z = |Z| e^{j\phi}$

$\vec{H}(z,t) = \frac{E_0}{|Z|} e^{-\alpha z} \cos(\omega t - \beta z - \phi)\,\hat{y}$

c) $\vec{S}(z,t) = \vec{E} \times \vec{H} = \frac{E_0^2}{|Z|} e^{-2\alpha z} \cos(\omega t - \beta z - \phi)\cos(\omega t - \beta z)\,\hat{z} =$

$= \frac{E_0^2}{|Z|} e^{-2\alpha z}\left[\cos(-\phi) + \cos(2\omega t - 2\beta z - \phi)\right]\frac{1}{2}\,\hat{z}$

d) $\langle \vec{S} \rangle = \frac{1}{T}\int_0^T \vec{S}\,dt = \frac{E_0^2}{2|Z|} e^{-2\alpha z} \cos(\phi)\,\hat{z}$

e) $\langle \vec{S} \rangle = \mathrm{Re}\left\{ \frac{\vec{E} \times \vec{H}^*}{2} \right\} = \mathrm{Re}\left\{ \frac{E_0^2 e^{-\alpha z 2} \cdot e^{-j\beta z} \cdot e^{j\beta z}}{2|Z| e^{-j\phi}}\,\hat{z} \right\} = \frac{E_0^2}{2|Z|} e^{-2\alpha z} \cos(\phi)\,\hat{z}$

f) $\frac{-\oint_S \langle \vec{S} \rangle\, ds}{V} = \frac{-\int_V \nabla\cdot(\langle \vec{S} \rangle)\, dV}{V} = -\nabla\cdot(\langle \vec{S} \rangle) = -\frac{\partial}{\partial z}\frac{E_0^2}{2|Z|} e^{-2\alpha z}\cos(\phi) = 2\alpha \langle \vec{S} \rangle$ $[W/m^3]$

g) $\frac{\sigma}{\omega\varepsilon} = 900$ $\qquad Z = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = 0.9935(1+j)$ $\qquad \alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \approx 0.4\pi^2$

$P_d\big|_{z=0} = 2.2\,E_0^2$

$\frac{\sigma}{\omega\varepsilon} = 1.044 \times 10^{12}$ $\qquad Z = 2{,}604 \times 10^{-4}(1+j)$ $\qquad \alpha \approx 15 \times 10^3$

$P_d\big|_{z=0} = 32 \times 10^6\,E_0^2$

10) a) $\vec{E}_1(z,t) = E_0 \cos(\omega t - k_z z)\,\hat{x} + E_0 \cos(\omega t + k_z z - \frac{\pi}{2})\,\hat{y}$

$\vec{E}_2(z,t) = E_0 \cos(\omega t - k_z z)\,\hat{x} + E_0 \cos(\omega t + k_z z + \frac{\pi}{2})\,\hat{y}$

b) $\hat{m} = \hat{z} \Rightarrow \hat{x} \times \hat{y} = \hat{z} \Rightarrow p = \frac{E_y}{E_x} = |p| e^{j\phi}$

1: $|p| = 1$, $\phi = -\frac{\pi}{2}$ $\Rightarrow$ PCR

2: $|p| = 1$, $\phi = \frac{\pi}{2}$ $\Rightarrow$ PCL

c) $\vec{E} = E_0(\hat{x} - j\hat{y}) e^{-jk_z z} + E_0(\hat{x} + j\hat{y}) e^{-jk_z z} =$

$= 2E_0 e^{-jk_z z}\,\hat{x}$

$\hookrightarrow$ Polarização Linear Horizontal

II $v_f = \frac{\omega}{k} \Rightarrow k_{z_1} = \frac{2\pi f}{1.6c}$ $\qquad k_{z_2} = \frac{2\pi f}{1.1 \cdot 1.6c}$

$\vec{E}_1(3,64) = E_0(\hat{x} - j\hat{y}) e^{-j11\pi}$

$\vec{E}_2(3,64) = E_0(\hat{x} + j\hat{y}) e^{-j10\pi}$

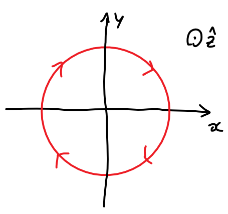
$\vec{E} = \vec{E}_1 + \vec{E}_2 = 2E_0 j\,\hat{y}$ $\Rightarrow$ Polarização Linear Vertical

11) $\vec{E}_1(z) = E_{0_1} e^{-\gamma z}(\hat{x} + j\hat{y})$

a) $\hat{m} = \hat{z} \Rightarrow \hat{x} \times \hat{y} = \hat{z}$

$p = \frac{E_y}{E_x} = |p| e^{j\phi}$

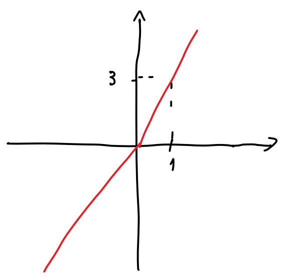
$|p| = 1$ $\qquad \phi = \frac{\pi}{2}$ $\Rightarrow$ PCL




$\vec{E}_2 = E_{0_2} e^{-\gamma z}(\hat{x} + 3\hat{y})$

$|p| = 3$

$\phi = 0$




b) $f_1 = 1\,kHz$ $\qquad \frac{\sigma}{\omega\varepsilon} = 6.75 \times 10^3$ $\qquad \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = 10.88 \times 10^{-3}$

$\varepsilon = 80\,\varepsilon_0$

$\mu = \mu_0$ $\qquad \frac{\sigma}{\omega\varepsilon} \gg 1$

$\sigma = 3 \times 10^{-2}$ $\qquad \Downarrow$

Bom condutor $\qquad Z = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = 362.76 \times 10^{-3}(1+j)$

c) $f_2 = 10\,GHz$ $\qquad \frac{\sigma}{\omega\varepsilon} = 0.33$ $\qquad \alpha = \mathrm{Re}\{\gamma\} = 311{,}8$

$\varepsilon = 80\varepsilon_0$

$\mu = \mu_0$ $\qquad \gamma = \alpha + j\beta = j\omega\sqrt{\varepsilon^c \mu}$ $\qquad \beta = \mathrm{Im}\{\gamma\} = 1.894 \times 10^3$

$\sigma = 15$ $\qquad \varepsilon^c = \varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right) =$ $\qquad Z = \sqrt{\frac{\mu}{\varepsilon_c}} = 40.5 + j6{,}7$

$= (707 - 238j) \times 10^{-12}$

12) a) $E(0,t) = E_0 \cos(\omega_0 t)\,\hat{x} + E_0 \cos(3\omega_0 t)\,\hat{x}$

$E(d,t) = E_0 \cos(\omega_0 t - k\frac{2\pi c_0}{\omega_0})\,\hat{x} + E_0 \cos(3\omega_0 t - k\frac{2\pi c_0}{\omega_0})\,\hat{x}$

$= E_0 \cos(\omega_0 t - 2\pi)\,\hat{x} + E_0 \cos(3\omega_0 t - 2\pi)\,\hat{x} =$

$= E(0,t)$ !

II $k(\omega_0) = k_1$ $\qquad \varepsilon(\omega_0) = 2{,}25\,\varepsilon_0$

$k(3\omega_0) = k_2$ $\qquad \varepsilon(3\omega_0) = 9\,\varepsilon_0$

b) $k_1 = \omega_0\sqrt{\mu\varepsilon} = 1.5\frac{\omega_0}{c_0}$ $\qquad v_{f_1} = \frac{\omega_0}{k_1} = \frac{c_0}{1.5} = 2 \times 10^8\,m/s$

$k_2 = 3\omega_0\sqrt{\mu\varepsilon} = 9\frac{\omega_0}{c_0}$ $\qquad v_{f_2} = \frac{3\omega_0}{k_2} = \frac{c_0}{3} = 1 \times 10^8\,m/s$

c) $E(0,t) = E_0 \cos(\omega_0 t)\,\hat{x} + E_0 \cos(3\omega_0 t)\,\hat{x}$

$E(d,t) = E_0 \cos(\omega_0 t - k_1\frac{2\pi c_0}{\omega_0})\,\hat{x} + E_0 \cos(3\omega_0 t - k_2\frac{2\pi c_0}{\omega_0})\,\hat{x} =$

$= E_0 \cos(\omega_0 t - 3\pi)\,\hat{x} + E_0 \cos(3\omega_0 t - 18\pi)\,\hat{x} =$

$= -E_0 \cos(\omega_0 t)\,\hat{x} + E_0 \cos(3\omega_0 t)\,\hat{x}$

13) a) $f = 6\,GHz$ $\quad v_f = \frac{\omega}{\beta} \approx \infty$

$v_g = \frac{d\omega}{d\beta} \approx 0$ (declive da reta tangente)

$f = 20\,GHz$ $\quad v_f \approx \pi \times 10^8$

$v_g \approx \frac{d\omega}{d\beta} \approx 2{,}5 \times 10^8$ (fig a desenhar)

b) $v_g(f = 7\,GHz) \approx 1{,}8 \times 10^8$ $\qquad v_g(f = 11\,GHz) \approx 2{,}3 \times 10^8$

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15) $\varepsilon = 4\varepsilon_0$ $\qquad \frac{\sigma}{\omega\varepsilon} = 0.225$

$\mu = \mu_0$

a) $\qquad \gamma = j\omega\sqrt{\mu\varepsilon^c} = 9.366 + 84.298j$

$\sigma = 0.1$

$\omega = 4\pi \times 10^9$ $\qquad \varepsilon^c = \varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)$ $\qquad \alpha = 9.366$

$\beta = 84.298$

$Z = \sqrt{\frac{\mu}{\varepsilon^c}} = 185 + j20.56\ \Omega = |Z| e^{j\theta_z}$

b) $\vec{E}(z) \to \vec{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z + \varphi_0)\,\hat{x}$

c) $\vec{H}(z) = \hat{m} \times \frac{\vec{E}}{Z} = \frac{E_0}{|Z|} e^{-\alpha z} e^{-j(\beta z - \varphi_0 + \theta_z)}\,\hat{y}$

$\vec{H}(z,t) = \frac{E_0}{|Z|} e^{-\alpha z}\cos(\omega t - \beta z + \varphi_0 - \theta_z)\,\hat{y}$

d) $\vec{S}(z,t) = \vec{E}(z,t) \times \vec{H}(z,t) = \frac{E_0^2}{|Z|} e^{-2\alpha z}\cos(\omega t - \beta z + \varphi_0 - \theta_z)\cos(\omega t - \beta z + \varphi_0)\,\hat{z}$

$= \frac{E_0^2}{2|Z|} e^{-2\alpha z}\left[\cos(2\omega t - 2\beta z + 2\varphi_0 - \theta_z) + \cos(\theta_z)\right]\hat{z}$

e) $\langle \vec{S} \rangle = \frac{1}{T}\int_0^T \vec{S}\,dt = \frac{E_0^2}{2|Z|} e^{-2\alpha z}\cos(\theta_z)\,\hat{z}$

f) $\frac{1}{2}\mathrm{Re}\{\vec{E} \times \vec{H}^*\} = \frac{1}{2}\mathrm{Re}\left\{ \frac{E_0 e^{-\alpha z} e^{-j\beta z} e^{j\varphi_0} \cdot E_0 e^{-\alpha z} e^{+j\beta z} e^{-j\varphi_0}}{|Z|} e^{j\theta_z} \right\}\hat{z}$

$= \frac{1}{2}\mathrm{Re}\left\{ \frac{E_0^2 e^{-2\alpha z}}{|Z|} e^{j\theta_z} \right\} = \frac{E_0^2 e^{-2\alpha z}}{2|Z|} \cdot \cos(\theta_z) = \langle \vec{S} \rangle$