



$$\begin{aligned} \nabla \times \vec{E} &= -\mu \frac{\partial \vec{H}}{\partial t} \\ \nabla \times \vec{H} &= \epsilon \frac{\partial \vec{E}}{\partial t} + \vec{j}_{ext} \\ \nabla \cdot \vec{D} &= j_{ext} \quad (\vec{D} = \epsilon \vec{E}) \\ \nabla \cdot \vec{B} &= 0 \quad (\vec{B} = \mu \vec{H}) \end{aligned}$$

$$\begin{aligned} \nabla &= \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \\ \nabla^2 &= \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \end{aligned}$$

$$\begin{aligned} \nabla \times \nabla \times \vec{A} &= \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \\ \nabla \cdot (\vec{A} \times \vec{B}) &= -\vec{A} \cdot \nabla \times \vec{B} + \vec{B} \cdot \nabla \times \vec{A} \end{aligned}$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x} \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) + \hat{y} \left(\frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \right) + \hat{z} \left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \right)$$

$$\begin{aligned} \vec{E} &= \vec{E}_0 \cos(\omega t - \vec{k} \cdot \vec{r} + \theta) \\ \vec{E} &= \vec{E}_0 e^{j(\omega t - \vec{k} \cdot \vec{r} + \theta)} = \vec{E}_0 e^{-j(\vec{k} \cdot \vec{r} - \theta)} \end{aligned}$$

$$\vec{k} = \|\vec{k}\| \cdot \hat{m} = k \cdot \hat{m}$$

$$\lambda = \frac{2\pi}{k} \quad f = \frac{\omega}{2\pi} \quad T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$c = \lambda f = \frac{1}{\sqrt{\mu \epsilon}} \quad v_f = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad v_g = \frac{d\omega}{dk}$$

$$k = \omega \sqrt{\mu \epsilon} = \frac{\omega}{c} \quad Z = \sqrt{\frac{\mu}{\epsilon}}$$

$$\vec{H} = \hat{m} \times \frac{\vec{E}}{Z} \quad \vec{E} = Z \vec{H} \times \hat{m}$$

$$\epsilon = \epsilon_0 \epsilon_r \quad \mu = \mu_0 \mu_r$$

$$v_f = c = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \cdot \mu_r \epsilon_r}} = \frac{c_0}{\sqrt{\mu_r \epsilon_r}} = \frac{c_0}{m}$$

$$m = \sqrt{\mu_r \epsilon_r}$$

$$k = \omega \sqrt{\mu \epsilon} = \frac{\omega}{c_0} \cdot m$$

$$Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \sqrt{\frac{\mu_r}{\epsilon_r}} = Z_0 \sqrt{\frac{\mu_r}{\epsilon_r}} \approx 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}}$$

$\mu_0 = 4\pi \times 10^{-7}$	$\epsilon_0 = 10^{-9}$	$c_0 = 3 \times 10^8$
$m = 10^{-3}$	$\mu = 10^{-6}$	$\epsilon = 10^{-9}$
$K = 10^3$	$M = 10^6$	$G = 10^9$
$\tau = 10^{12}$	$P = 10^{16}$	

$$\begin{aligned} jk &= \gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \\ &= j\omega\sqrt{\mu\epsilon} \sqrt{1 - j\frac{\sigma}{\omega\epsilon}} = j\omega\sqrt{\epsilon^c} \mu \end{aligned}$$

$$Z = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon^c}} \quad \beta = \frac{2\pi}{\lambda}$$

$$\epsilon^c = \epsilon + \frac{\sigma}{j\omega} = \epsilon \left(1 + \frac{\sigma}{j\omega\epsilon} \right) = \epsilon_r \epsilon_0 \left(1 + \frac{\sigma}{j\omega\epsilon_r \epsilon_0} \right)$$

$$\rightarrow \sigma = 0$$

$$\alpha = 0 \quad \beta = \frac{2\pi}{\lambda} = \omega \sqrt{\mu \epsilon} = \frac{\omega}{c}$$

$$c = \frac{c_0}{\sqrt{\epsilon_r \mu_r}} \quad \lambda = \frac{\lambda_0}{\sqrt{\epsilon_r \mu_r}} \quad Z = \sqrt{\frac{\mu}{\epsilon}} = Z_0 \sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\rightarrow \sigma \ll \omega\epsilon \Rightarrow \tan(\theta_p) = \frac{\sigma}{\omega\epsilon} \ll 1$$

$$\sqrt{1-x} \approx 1 - \frac{x}{2} \quad (x \ll 1)$$

$$\beta \approx \frac{2\pi}{\lambda} = \omega \sqrt{\mu \epsilon} = \frac{\omega}{c} \quad \alpha \approx \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} = \beta \frac{\tan(\theta_p)}{2}$$

$$c = \frac{c_0}{\sqrt{\epsilon_r \mu_r}} \quad \lambda = \frac{\lambda_0}{\sqrt{\epsilon_r \mu_r}} \quad Z \approx \sqrt{\frac{\mu}{\epsilon}} (1 + j \frac{\sigma}{2\omega\epsilon})$$

$$\rightarrow \sigma \gg \omega\epsilon \Rightarrow \tan(\theta_p) = \frac{\sigma}{\omega\epsilon} \gg 1$$

$$\alpha = \beta = \frac{1}{\delta} \approx \sqrt{\frac{\omega\mu\sigma}{2}} \quad Z \approx \sqrt{\frac{\omega\mu}{2\sigma}} (1 + j)$$

$$\theta_E - \theta_H = \pm \{Z\} \quad \delta = \frac{1}{\alpha}$$

$$\vec{E} = \vec{E}_0 e^{-j\vec{k} \cdot \vec{r} + \alpha \cdot \vec{r}} = E_1 \hat{x}_1 + E_2 \hat{x}_2 \quad \hat{x}_1 \times \hat{x}_2 = \hat{m}$$

$$P = \frac{E_2}{E_1} = |P| e^{j\phi}$$

$$i) |P| = 1 \wedge \phi = \pm \frac{\pi}{2} \Rightarrow \begin{matrix} + \rightarrow PCL \\ - \rightarrow PCR \end{matrix}$$

$$ii) p=0 \vee p=\infty \vee \phi=0 \vee \phi=\pi \Rightarrow PL$$

$$iii) PE \begin{cases} 0 < \phi < \pi \rightarrow PEL \\ -\pi < \phi < 0 \rightarrow PED \end{cases}$$

$$\begin{aligned} \hat{x} \times \hat{x} &= 0 & \hat{y} \times \hat{x} &= -\hat{z} & \hat{z} \times \hat{x} &= \hat{y} \\ \hat{x} \times \hat{y} &= \hat{z} & \hat{y} \times \hat{y} &= 0 & \hat{z} \times \hat{y} &= -\hat{x} \\ \hat{x} \times \hat{z} &= -\hat{y} & \hat{y} \times \hat{z} &= \hat{x} & \hat{z} \times \hat{z} &= 0 \end{aligned}$$

$$\begin{aligned} \hat{e}_{11} \times \hat{e}_{11} &= \hat{m}_i & 1+R_I &= T_{\perp} \\ \hat{e}_{11} \times \hat{e}_{11} &= \hat{m}_n & 1+R_{II} &= T_{\parallel} \frac{\cos(\theta_t)}{\cos(\theta_i)} \\ \hat{e}_{11}^+ \times \hat{e}_{11}^+ &= \hat{m}^t & & \end{aligned}$$

$$\sigma = 0: \frac{d\mathcal{E}_{EM}}{dt} = -P_{\Sigma}(t) + P_{im}(t)$$

$$\nabla \cdot (\vec{E} \times \vec{H}) = -\vec{E} \cdot \vec{j}_{ext} - \frac{\partial}{\partial t} W_{EM}$$

$$W_{EM} = \frac{1}{2} [\epsilon \vec{E}^2 + \mu \vec{H}^2]$$

$$\int_V \nabla \cdot (\vec{E} \times \vec{H}) dV = -\int_V \vec{E} \cdot \vec{j}_{ext} dV - \frac{d}{dt} \int_V W_{EM} dV$$

$$\mathcal{E}_{EM} = \int_V W_{EM} dV$$

$$P_{im}(t) = -\int_V \vec{E} \cdot \vec{j}_{ext} dV$$

$$P_{\Sigma}(t) = \int_V \nabla \cdot (\vec{E} \times \vec{H}) dV = \int_{\Sigma} \vec{S} \cdot \hat{n} ds$$

$$\vec{S} = \vec{E} \times \vec{H} \quad S(t) = E(t) \times H(t)$$

$$\vec{S}_{av} = \frac{1}{2} \text{Re} \{ \vec{E} \times \vec{H}^* \}$$

$$\vec{S}_{av} = \frac{1}{2} \text{Re} \left\{ \frac{1}{Z} |\vec{E}|^2 \cdot \hat{m} \right\}$$

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad T = 1 + R = \frac{2Z_2}{Z_1 + Z_2}$$

$$E_o^{ref} = R E_o^{inc} \quad E_o^{tr} = T E_o^{inc}$$

$$\frac{S^{ref}}{S^{inc}} = R^2 \quad \frac{S^{tr}}{S^{inc}} = 1 - R^2$$

$$\left\{ \begin{aligned} \theta_i &= \theta_r & |R_T| = 1 \rightarrow \text{num}(\theta_{lm}) = m_2 \pm 1 \\ \text{num}(\theta_t) m_2 &= \text{num}(\theta_i) m_1 \end{aligned} \right.$$

$$\theta_t = 90^\circ \Rightarrow \theta_{lm} = \theta_{out} = \arccos \left(\frac{m_2}{m_1} \right)$$

$$\cos(\theta_B) = \frac{1}{\sqrt{\left(\frac{m_2}{m_1}\right)^2 + 1}} = \frac{1}{\sqrt{1 + \frac{\epsilon_2}{\epsilon_1}}}$$

$$t_g(\theta_B) = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$R_{\perp} = \frac{Z_2 \cos(\theta_i) - Z_1 \cos(\theta_t)}{Z_2 \cos(\theta_i) + Z_1 \cos(\theta_t)} \quad \sigma = 0$$

$$R_{\parallel} = \frac{Z_1 \cos(\theta_i) - Z_2 \cos(\theta_t)}{Z_1 \cos(\theta_i) + Z_2 \cos(\theta_t)} \quad \sigma = 0$$

$$\vec{E}^{inc} = (E_{10}^i \hat{e}_{11} + E_{10}^i \hat{e}_{11}) e^{-j\vec{k}_i \cdot \vec{r} + \alpha^i \cdot \vec{r}}$$

$$\vec{E}^{ref} = (R_{II} E_{10}^i \hat{e}_{11} + R_{\perp} E_{10}^i \hat{e}_{11}) e^{-j\vec{k}_r \cdot \vec{r} + \alpha^r \cdot \vec{r}}$$

$$\vec{E}^{tr} = \left(\frac{Z_2}{Z_1} (1+R_{II}) E_{10}^i \hat{e}_{11}^+ + (1+R_{\perp}) E_{10}^i \hat{e}_{11} \right) e^{-j\vec{k}_t \cdot \vec{r} + \alpha^t \cdot \vec{r}}$$

$$P_i = \frac{E_{10}^i}{E_{10}^i} \quad P_r = \frac{E_{10}^r}{E_{10}^i} = \frac{R_{\perp}}{R_{II}} P_i$$

$$P_t = \frac{1+R_{\perp}}{Z_1} P_i \quad SWR = \frac{1+|R_I|}{1-|R_I|} = \frac{|E|_{max}}{|E|_{min}}$$