$$\sin(\phi_{Lim}) = \frac{n_2}{n_1}$$

$$AN = \sin(\theta_a) = \sqrt{n_1^2 - n_2^2}$$

$$\Delta = \frac{n_1^2 - n_2^2}{2 n_1^2} = \frac{AN^2}{2 n_1^2}$$

$n_1 \approx n_2$

$$\hookrightarrow \Delta \approx \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1}$$

$\Delta \ll 1$

$$k_0 = \frac{\omega}{c_0} = \frac{2\pi}{\lambda_0}$$

$$u^2 + k_z^2 = k_1^2 = n_1^2 k_0^2$$

$$-w^2 + k_z^2 = k_2^2 = n_2^2 k_0^2$$

$$u^2 + w^2 = k_0^2 (n_1^2 - n_2^2)$$

$$U = a u \qquad W = a w$$

$$V = \sqrt{U^2 + W^2} = a\sqrt{u^2 + w^2}$$

$$V = a k_0 \sqrt{n_1^2 - n_2^2} = a k_0 AN = a n_1 k_0 \sqrt{2\Delta}$$

$$b = \frac{W^2}{V^2} = 1 - \frac{U^2}{V^2}$$

$$b = \frac{\left(\frac{k_z}{k_0}\right)^2 - n_2^2}{n_1^2 - n_2^2} \qquad \frac{k_z}{k_0} = n$$

$$U = \frac{(1+\sqrt{2})V}{1 + (4 + V^4)^{1/4}}$$

$1.5 < V < 2.5$

$$\hookrightarrow U = \sqrt{V^2 - (1.1428V - 0.996)^2}$$

$V \gg 1$

$$\hookrightarrow \quad M \approx \frac{V^2}{2}$$

LP_{01} - HE_{11} - $V_c = 0$

LP_{11} - TE_{01}, TM_{01}, HE_{21} - $V_c = 2.405$

LP_{21} - EH_{11}, HE_{31} - $V_c = 3.832$

$\Delta \ll 1$

$$\Delta t_{inter} = \frac{L}{c_0}\frac{n_1^2}{n_2} - \frac{L}{c_0} n_1 \approx \frac{L}{c_0} n_1 \Delta$$

$$\Delta t_{intra} = L |M| \Delta\lambda$$

$$B = \frac{1}{2\Delta t}$$

$$A = \frac{P(0)}{P(L)} = e^{2\alpha L}$$

$$A_{dB} = 10 \log_{10}(A) = 20 \alpha L \log_{10}(e)$$

DEH

$$\bar{E}_r = \frac{Z \bar{I}_0 L}{2\pi r}\left(1 + \frac{1}{jkr}\right)\frac{e^{-jkr}}{r}\cos(\theta)$$

$$\bar{E}_\theta = j\frac{Z k \bar{I}_0 L}{4\pi}\left(1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right)\frac{e^{-jkr}}{r}\sin(\theta)$$

$$\bar{H}_\varphi = j\frac{k \bar{I}_0 L}{4\pi}\left(1 + \frac{1}{jkr}\right)\frac{e^{-jkr}}{r}\sin(\theta)$$

$$\bar{E}_\varphi = \bar{H}_\theta = \bar{H}_r = 0$$

$$P_r = \frac{\pi}{3} Z \left(\frac{L}{\lambda}\right)^2 |I_0|^2 \qquad R_r = \frac{2\pi}{3} Z \left(\frac{L}{\lambda}\right)^2$$

ZD:

1. $r \gg \lambda_0$
2. $r \gg L$
3. $r > \frac{2L^2}{\lambda}$

$$\bar{E}_r = \bar{E}_\varphi = \bar{H}_\theta = \bar{H}_r = 0$$

$$\bar{E}_\theta = j\frac{Z \bar{I}_0 L}{2\lambda} \cdot \frac{e^{-jkr}}{r}\sin(\theta)$$

$$\bar{H}_\varphi = \frac{\bar{E}_\theta}{Z}$$

$$\vec{S}_{av} = \frac{1}{2}\mathrm{Re}\{\vec{E} \times \vec{H}^*\} = \frac{|E|^2}{2Z}\hat{r} = \frac{Z|H|^2}{2}\hat{r}$$

$$U = r^2 \vec{S}_{av} \qquad d\Omega = \sin(\theta)\, d\theta\, d\varphi$$

$$P_r = \oint U\, d\Omega = \int_0^{2\pi}\int_0^{\pi} \vec{S}_{av}\, r^2 \sin(\theta)\, d\theta\, d\varphi$$

$$P_{in} = \frac{1}{2} R_A |I_0|^2 \qquad P_r = \frac{1}{2} R_r |I_0|^2$$

$$P_p = \frac{1}{2} R_p |I_0|^2 \qquad R_A = R_r + R_p$$

$$g(\theta, \varphi) = \frac{U}{U_{av}} = \frac{4\pi U(\theta,\varphi)}{P_r}$$

$$\alpha_{-3dB} = HPBW \Rightarrow \frac{U}{U_{max}} = \frac{1}{2}$$

$$D = \frac{U_{max}}{U_{av}} = \frac{4\pi U_{max}}{P_r} \approx \frac{4\pi}{\alpha_H \alpha_V} \qquad G_p = \frac{4\pi U_{max}}{P_i}$$

$$\eta = \frac{P_r}{P_p + P_r} = \frac{P_r}{P_i} = \frac{R_r}{R_r + R_p} = \frac{R_r}{R_A} = \frac{G_p}{D}$$

DL: $\bar{I}(z) = I_m \sin\left(k_0\left(\frac{L}{2} - |z|\right)\right)$

$$I_0 = I_m \sin\left(\frac{k_0 L}{2}\right)$$

$$P(\theta) = \frac{\cos\left(k\frac{L}{2}\cos(\theta)\right) - \cos\left(k\frac{L}{2}\right)}{\sin(\theta)}$$

ZD: $\bar{E}_\theta = j\frac{Z \bar{I}_m}{2\pi} \cdot \frac{e^{-jkr}}{r} \cdot P(\theta)$

$$\bar{E}_\theta = Z \bar{H}_\varphi$$

$$g(\theta) = \frac{Z}{\pi R_r}\left|\frac{I_m}{I_0}\right|^2 |P(\theta)|^2$$

- $L \ll \lambda_0$
  - $R_r = \frac{1}{4} R_{r\,DEH}$
  - $R_p = \frac{1}{3} R_{p\,DEH}$
  - $I = I_0\left(1 - \frac{2|z|}{L}\right)$
  - $\bar{E}_\theta = j\frac{Z(m+1)\bar{I}_0}{4} \cdot \frac{L}{\lambda} \cdot \frac{e^{-jkr}}{r} \cdot \sin(\theta)$

- $L = \frac{\lambda}{2}$
  - $g = 1.64\left|\frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin(\theta)}\right|^2$
  - $R_r = 73.1\,\Omega$
  - $I = I_0 \sin\left(\frac{\pi}{2}\left(1 - \frac{2|z|}{L}\right)\right)$

- $L = \lambda$
  - $R_r = \infty$ ($\sim M\Omega$)
  - $D = 3{,}90$ dB

- $L = \frac{3\lambda}{2}$
  - $R_r = 105.3\lambda$

$$R_a = \frac{R_r}{\eta} \qquad X_a \approx -Z_L \,\mathrm{cotg}\left(\frac{k_0 L}{2}\right)$$

$$c_0 = 3 \times 10^8 \qquad \mu_0 = 4\pi \times 10^{-7} \qquad \varepsilon_0 = \frac{10^{-9}}{36\pi}$$

$m - 10^{-3}$	$k - 10^{3}$	$\beta = \frac{2\pi}{\lambda}$	$\int_0^\pi \sin^3(\theta)\, d\theta = \frac{4}{3}$
$\mu - 10^{-6}$	$M - 10^{6}$	$v_f = \frac{\omega}{\beta}$	
$n - 10^{-9}$	$G - 10^{9}$		$Z = \sqrt{\frac{\mu}{\varepsilon}}$
$p - 10^{-12}$	$T - 10^{12}$	$v_g = \frac{d\omega}{d\beta}$	
$f - 10^{-15}$	$P - 10^{15}$	$c = \lambda f$	$k = \frac{\omega}{c_0}\sqrt{\mu\varepsilon}$