1. a) $AN = \sqrt{n_1^2 - n_2^2} = 0{,}1706$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{AN^2}{2n_1^2} = 6{,}83 \times 10^{-3}$$

$\sin(\theta_a) = AN \Leftrightarrow \theta_a = \sin^{-1}(AN) = 9.82^\circ$

b) $V = a\,k_0 AN = a \cdot \frac{2\pi}{\lambda_0} \cdot AN = 63{,}05$

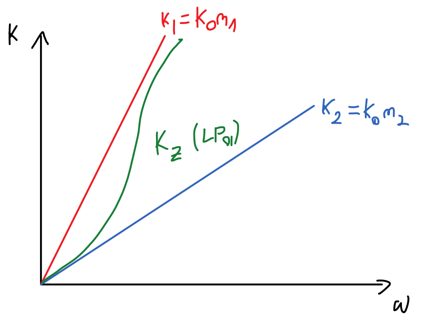
$k_0 = \frac{2\pi}{\lambda_0}$

Como $V > 2{,}405$, temos regime multimodal com aprox: $\frac{V^2}{2} \approx 1988$ modos!

c) O modo fundamental é sempre HE_{11} (LP_{01})

$k_z = \sqrt{k_1^2 - u^2} = \sqrt{k_0^2 n_1^2 - u^2}$

$u = \frac{U}{a} \approx \frac{(1+\sqrt{2})V}{a(1+\sqrt[4]{4+V^4})}$ onde $V = a\,\frac{2\pi \boxed{f}}{c_0} AN$ depende aqui



d) $n_1 > n_2$ para haver reflexão total

Se $n_1 \gg n_2$ temos dispersão, é mau!

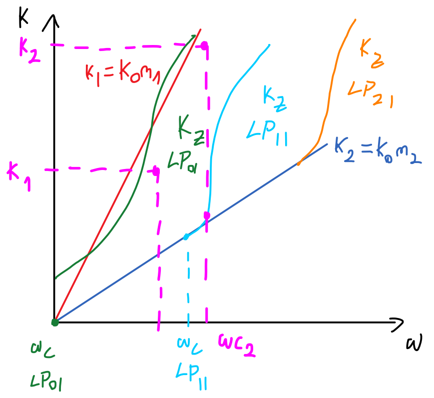
2. a) $V_1 < 2{,}405 \rightarrow$ Unimodal, LP_{01} (HE_{11})

$2{,}405 < V_2 < 3{,}832 \rightarrow$ Multimodal, LP_{01} (HE_{11}) e LP_{11} ($TE_{01} + TM_{01} + HE_{21}$)

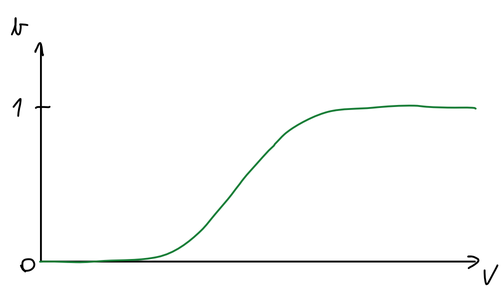
b) $V = 2{,}405$

$V = a\,k_0\sqrt{n_1^2 - n_2^2} \Leftrightarrow \lambda_{min} = \frac{2\pi \cdot a \cdot \sqrt{n_1^2 - n_2^2}}{V} = 1{,}493\ \mu m$

c)
d)
e)



f)



g) $V_1 = 2{,}4$

$\frac{n_1}{n_2} = 1{,}01$

$\Delta = 0{,}99\%$

$\lambda = 1{,}0657\ \mu m$

$U = 1{,}670$

$W = 1{,}7236$

$b = 0{,}5157$

3. a) $V_{Max} = a\,k_0\sqrt{n_1^2 - n_2^2} = a\,n_1 k_0\sqrt{2\Delta} \Leftrightarrow \lambda_{min} = \frac{2\pi \cdot a \cdot n_1\sqrt{2\Delta}}{V_{Max}}$

$f_{Max} = \frac{c_0}{\lambda_{min}} = \frac{V_{Max} \cdot c_0}{2\pi \cdot a \cdot n_1 \cdot \sqrt{2\Delta}} = 266{,}32\ THz$

$f_{min} = 0$

b) $V = a\,k_0 n_1\sqrt{2\Delta} \Leftrightarrow V = 2{,}084$

$V < 2{,}405 \Rightarrow$ Unimodal

Pela aproximação de Rudolph-Neumann ($1{,}5 < V < 2{,}5$)

$U \approx \sqrt{V^2 - (1{,}1428V - 0{,}996)^2} = 1{,}5566$

$U = a\,u = a\sqrt{n_1^2 k_0^2 - k_z^2} \Leftrightarrow k_z = \sqrt{n_1^2 k_0^2 - \left(\frac{U}{a}\right)^2} = 7{,}126 \times 10^6\ rad/m$

$V_{ph} = \frac{\omega}{k_z} = 2{,}035 \times 10^8\ m/s$

c) • $V = 1{,}1 \rightarrow f = \frac{V \cdot c_0}{2\pi \cdot a \cdot n_1 \cdot \sqrt{2\Delta}} = 121{,}81\ THz$

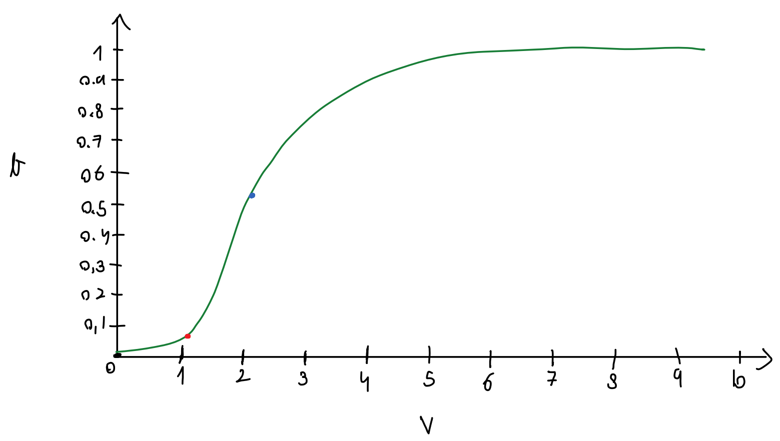
$\rightarrow U \approx \frac{(1+\sqrt{2})V}{1+\sqrt[4]{4+V^4}} = 1{,}0601$

$\rightarrow b = 1 - \frac{U^2}{V^2} = 0{,}0886$

• $V = 2{,}35 \rightarrow f = \frac{V \cdot c_0}{2\pi \cdot a \cdot n_1 \cdot \sqrt{2\Delta}} = 260{,}23\ THz$

$\rightarrow U \approx \sqrt{V^2 - (1{,}1428V - 0{,}996)^2} = 1{,}6334$

$\rightarrow b = 1 - \frac{U^2}{V^2} = 0{,}5168$



d) b: Constante de propagação normalizada

V: frequencia normalizada

e) —

f) $\Delta\lambda = 5\ nm$

$\lambda = 1{,}3\ \mu m \Rightarrow M \approx -8\ ps/Km\,nm$

$\Delta t_{intra} = L|M|\Delta\lambda$

$B_{Max} = \frac{1}{2L|M|\Delta\lambda} = 1{,}25\ Gb/s$

$\Delta t_{inter} = 0$! (é monomodal)

4. $a = 3\,\mu m$ $\quad \lambda_1 = 1300\,nm$ $\quad \lambda_2 = 1550\,nm$

$n_1 = 1{,}486$ $\quad M_1 = -5\,ps/Km \cdot nm$ $\quad M_2 = -25\,ps/Km \cdot nm$

$\phi_{lim} = 82{,}7^{o}$

a) $n_2 = \sin(\phi_{lim})\, n_1 = 1{,}474$ $\qquad AN = \sqrt{n_1^2 - n_2^2} = 1{,}888$

$\Delta = \frac{AN^2}{2n_1^2} = 8{,}07 \times 10^{-3}$

$AN = 1{,}888 \Rightarrow$ Caracteriza o cone de aceitação

$\Delta = 8{,}07 \times 10^{-3} \Rightarrow$ Diferença relativa entre a bainha e o núcleo

b) $V_1 = n_1 K_{01}\, a\sqrt{2\Delta} = n_1 \frac{2\pi}{\lambda_1} a\sqrt{2\Delta} = 2{,}7378$ $\qquad V_1 > 2{,}405 \rightarrow$ Multimodal, LP_{01} e LP_{11}

$V_2 = n_1 K_{02}\, a\sqrt{2\Delta} = n_1 \frac{2\pi}{\lambda_2} a\sqrt{2\Delta} = 2{,}2962$ $\qquad V_2 < 2{,}405 \rightarrow$ Monomodal, LP_{01}

$U_1 = \frac{(1+\sqrt{2})V}{1+(4+V^4)^{1/4}} = 1{,}7461$ $\qquad W_1 = \sqrt{V_1^2 - U_1^2} = 2{,}1087$

$U_2 = \sqrt{V^2 - (1{,}1428V - 0{,}996)^2} = 1{,}6192$ $\qquad W_2 = \sqrt{V_2^2 - U_2^2} = 1{,}6281$

c) —

d) $\Delta\lambda = 5\,nm$ $\quad$ Caso 1:

$L = 20\,km$

$\Delta t_{inter} \approx \frac{L}{C_0} n_1 \Delta \approx 800\,nS$ $\qquad \Delta t_{total} = \Delta t_{inter} + \Delta t_{intra} = 800{,}5\,nS$

$\Delta t_{intra} = L\,|M_1|\,\Delta\lambda = 0{,}5\,nS$

$B_{max} = \frac{1}{2\Delta t} = 624{,}6\,Kb/s$

Caso 2: $\Delta t_{inter} = 0$ (É monomodal)

$\Delta t_{intra} = L\,|M_2|\,\Delta\lambda = 2500\,pS$

$B_{max} = \frac{1}{2\Delta t} = 200\,Mb/s$

e) Caso 1: Menor B_{max} e mais dispersão e atenuação

Caso 2: Maior B_{max} e menos dispersão e atenuação mas muito difícil de implementar

5. $a = 25\,\mu m$ $\quad$ a) $AN = \sqrt{2\Delta n_1^2} = 0{,}2051$

$n_1 = 1{,}45$ $\qquad \theta_a = \sin^{-1}(AN) = 11{,}83^{o}$

$\Delta = 1\%$

$\lambda = 0{,}85\,\mu m$

b) $V = n_1 K_0\, a\sqrt{2\Delta} = 37{,}895$

$M \approx \frac{V^2}{2} = 718$ modos $\qquad$ É multimodal

c) —

d) $\Delta t_{inter} \approx \frac{L}{C_0} n_1 \Delta$

$\frac{\Delta t_{inter}}{L} \approx \frac{n_1 \Delta}{C_0} = 48{,}3\,nS/Km$

e) $\Delta t_{inter} = 241{,}5\,nS$

Δt_{intra} é muito pequeno comparado a Δt_{inter}

$B_{max} = \frac{1}{2\Delta t_{inter}} = 2{,}07\,Mb/s$

$LB = B = 2{,}07\,MHz$

f) $n_1 = 1{,}5$ $\quad V = K_0\, a\sqrt{n_1^2 - n_2^2} = 398 \Rightarrow$ Multimodal

$n_2 = 1{,}4$ $\quad$ Como é Multimodal a dispersão intermodal

$a = 100\,\mu m$ $\quad$ é mais forte!

$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \not\ll 1 \rightarrow \Delta t = \frac{L}{C_0}\left(\frac{n_1^2}{n_2} - n_1\right)$

$B = \frac{1}{2\Delta t_{inter}} \Rightarrow BL = \frac{C_0}{2\left(\frac{n_1^2}{n_2} - n_1\right)} = 1{,}4\,Mb \cdot Km/s$

$2 \cdot L = 1{,}4 \Rightarrow L = 0{,}7\,Km = 700\,m$

6. a) $\Delta = 1\%$

$V = a\, n_1 K_0 \sqrt{2\Delta} = 3{,}3938$

$V > 2{,}405 \Rightarrow$ Multimodal

b) $AN = \sqrt{2 n_1^2 \Delta} = 0{,}2093$

$n_2 = \sqrt{n_1 - AN^2} = 1{,}465$

$\phi_{lim} = \arcsin\left(\frac{n_2}{n_1}\right) = 81{,}87^{o}$

c) $\Delta t \approx \frac{L}{C_0} n_1 \Delta$

$BL = \frac{L}{2\Delta t} \Rightarrow BL = \frac{C_0}{2n_1\Delta} = 10{,}135\,Mb \cdot Km/s$

d) $LB = B \Rightarrow LB = 10{,}135\,MHz$

e) $V < 2{,}405$ $\quad \Rightarrow \quad a < \frac{2{,}405}{n_1 K_0 \sqrt{2\Delta}} \Rightarrow a_{max} = 2{,}835\,\mu m$

$V = a\, n_1 K_0 \sqrt{2\Delta}$

$\hookrightarrow$ Monomodal $\Rightarrow$ Menos dispersão

f) $V = 2{,}405$

$U = \sqrt{V^2 - (1{,}1428V - 0{,}996)^2} = 1{,}6471 \rightarrow u = \frac{U}{a} = 5{,}81 \times 10^{6}\,Rad/m$

$W = \sqrt{V^2 - U^2} = 1{,}7524 \rightarrow w = \frac{W}{a} = 6{,}182 \times 10^{5}\,Np/m$

7. a) $n_2 = 1{,}4686$

$\sin(\phi_{lim}) = \frac{n_2}{n_1} \Leftrightarrow n_1 = \frac{n_2}{\sin(\phi_{lim})} = 1{,}4882$

$AN = \sqrt{n_1^2 - n_2^2} = 0{,}2405$

$\Delta = \frac{AN^2}{2n_1^2} = 0{,}01306$

b) B_{max} com $L = 30\,Km$

$V = a K_0\, AN = 17{,}435 \Rightarrow$ Regime multimodal

inter

$\Delta t = \frac{L}{C_0} \cdot n_1 \cdot \Delta = 1{,}943\,\mu S$

intra

$\Delta t = L\,|M|\,\Delta\lambda = 78\,nS$ $\qquad \Delta t_{inter} \gg \Delta t_{intra} \rightarrow \Delta t_{total} \approx \Delta t_{inter}$

$B_{max} = \frac{1}{2\Delta t_{total}} = 257{,}304\,Kb/s$

c) Com código RZ $\rightarrow LB = B_{max}$

Logo $\left\lfloor \frac{LB}{4K} \right\rfloor = N^{o}$ Canais $= 64$

Para aumentar o N^{o} de canais temos de aumentar a LB

Logo temos de diminuir a dispersão por isso podemos reduzir o raio da fibra (a) para termos uma fibra monomodal.

d) $V = a K_0 AN \Leftrightarrow a = \frac{V}{K_0 AN}$ $\quad$ onde $V = 2{,}405$

$a = 2{,}0691\,\mu m$

e)

	V	U	u	b
$0{,}1 f_0$	0,22	0,2199	$14{,}6 \times 10^{3}$	171×10^{-6}
f_0	2,2	1,5922	106×10^{3}	0,476
$5 f_0$	11	2,2129	147×10^{3}	0,959

$u = \frac{U}{a}$ $\qquad b = 1 - \frac{U^2}{V^2}$

$V = a K_0 AN$ $\qquad 0{,}1 f_0 \Rightarrow V = 0{,}22$

$\hookrightarrow V = \frac{a \cdot 2\pi \cdot AN}{c} f$ $\qquad 5 f_0 \Rightarrow V = 11$

Se $1{,}5 < V < 2{,}5$

$U = \sqrt{V^2 - (1{,}1428V - 0{,}996)^2}$

Se não:

$U = \frac{(1+\sqrt{2})V}{1+(4+V^4)^{1/4}}$

8. a) $\phi_{lim} = \arcsin\left(\frac{n_2}{n_1}\right) \Rightarrow \frac{n_2}{n_1} = 0.9945$

$\Delta = \frac{n_1^2 - n_2^2}{2 n_1^2} = \frac{1}{2}\left(1 - \left(\frac{n_2}{n_1}\right)^2\right) = 5.46 \times 10^{-3}$

$B = \frac{1}{2\Delta t} \rightarrow BL = \frac{c_0}{2\Delta n_1} \Leftrightarrow n_1 = \frac{c_0}{2BL\Delta} = 1.445$

$\Delta = \frac{AN^2}{2n_1^2} \Leftrightarrow AN = \sqrt{2 n_1^2 \Delta} = 0.1511$

$\theta_a = \sin^{-1}(AN) = 8.69°$

b) $\Delta t = \frac{L}{c_0} n_1 \Delta = 7.895 \times 10^{-7}$ s

c) $V = a \frac{2\pi}{\lambda} AN$

$V_c = 2.405$ $\quad \frac{2.405}{2\pi AN} \lambda = a \Leftrightarrow a = 3.294\ \mu m$

$d = 2a = 6.588\ \mu m$

$\Delta\lambda = 0.2$ ps / km · nm

É unimodal ⇒ intra é mais importante

$B = \frac{1}{2\Delta t} \Leftrightarrow B = \frac{1}{2L|M|\Delta\lambda} \Leftrightarrow \Delta\lambda = \frac{1}{2L|M|B} = 8.(3)$ nm

$\Delta t = L|M|\Delta\lambda$

d)

9. a) $a = 20\lambda_0$ $\quad AN = \sin(\theta_a) = 0.1718$

$\Delta = \frac{0.68}{100}$

$\theta_a = 9.89°$ $\quad \Delta = \frac{1}{2}\left(1 - \left(\frac{n_2}{n_1}\right)^2\right) \Leftrightarrow \frac{n_2}{n_1} = \sqrt{1 - 2\Delta} = 0.9932$

$\phi_{lim} = \arcsin\left(\frac{n_2}{n_1}\right) = 83.30°$

b) $V = a \frac{2\pi}{\lambda_0} \cdot AN = 40\pi AN = 21.584$ $\quad V > 2.405 \Rightarrow$ Multimodal

c) $M \approx \frac{V^2}{2} \approx 233$

d) $\Delta t \approx \frac{c_0}{L} n_1 \Delta = \frac{L}{c_0}\sqrt{\frac{AN^2 \Delta}{2}} = 667.67$ ns

$n_1 = \sqrt{\frac{AN^2}{2\Delta}}$

e) $B = \frac{1}{2\Delta t} \Leftrightarrow BL = \frac{L}{2\Delta t} = 14.98$ Mb · Km / s

f) $B = \frac{BL}{L} = 748.87$ Kb · s $\Rightarrow LB = 748.87$ KHz

g) $V = a \frac{2\pi}{\lambda} AN \Leftrightarrow \frac{2.405 \cdot \lambda}{2\pi AN} = a = 3.454\ \mu m$

Monomodal ⇒ V < 2.405

10. a) $LP_{01} \rightarrow V_c = 0$

$LP_{11} \rightarrow V_c = 2.405$

$LP_{21} \rightarrow V_c = 3.832$

b) $V_{min} = 0 \rightarrow f_{min} = 0$

$V_{max} = 2.405$

$2a = 5\ \mu m \Leftrightarrow a = 2.5\ \mu m$

$V = a k_0 n_1 \sqrt{2\Delta} \Leftrightarrow \lambda_0 = \frac{a \cdot 2\pi \cdot n_1\sqrt{2\Delta}}{V} = 1.126\ \mu m$

$f = \frac{c}{\lambda_0} = 266.32$ THz

$f \in [0, 266.32 \text{ THz}]$

c) $\lambda_0 = 1.3\ \mu m$

$V = a \frac{2\pi}{\lambda_0} n_1 \sqrt{2\Delta} = 2.084$

$U = \sqrt{V^2 - (1.1428V - 0.996)^2} = 1.5566$ rad

$u = \frac{U}{a} = 6.227 \times 10^5$ rad / m

$W = \sqrt{V^2 - U^2} = 1.3855$ Np

$w = \frac{W}{a} = 5.542 \times 10^5$ Np/m

$u^2 + k_z^2 = n_1^2 k_0^2$

$k_z = \sqrt{n_1^2 k_0^2 - u^2} = 7.126 \times 10^6$ rad / m

$k_z = \frac{\omega}{v_f} \Leftrightarrow v_f = \frac{\omega}{k_z} = \frac{2\pi f}{k_z} = \frac{\frac{2\pi c}{\lambda_0}}{k_z} = \frac{2\pi c}{\lambda_0 k_z} = 203.475 \times 10^6$ m/s

d) $b = 1 - \frac{U^2}{V^2} = 0.442$

e) —

f) $V < 2.405 \Rightarrow$ Monomodal

Só temos intra

$\Delta t = L|M|\Delta\lambda = 1.256$ b/s

g) $2a' = 50\ \mu m \Rightarrow a' = 10a$

$2a = 5\ \mu m$

$V' = a' k_0 n_1 \sqrt{2\Delta} \Rightarrow V' = 10V = 20.840 \rightarrow$ Regime multimodal

inter:

$\Delta t = \frac{L}{c_0} n_1 \Delta = 335$ ns

intra: $\quad \Delta t_{inter} >> \Delta t_{intra} \Rightarrow \Delta t_{total} \approx \Delta t_{inter}$

$\Delta t = L|M|\Delta\lambda = 400$ ps

$B = \frac{1}{2\Delta t} \approx 1.5$ Mb/s

11. a) $n_1 = 1.48$ $\quad \lambda_0 = 850$ nm

$\Delta = 0.5\%$ $\quad \Delta\lambda = 35$ nm

$a = 25\ \mu m$ $\quad M = -10$ ps/nm · Km

$P_e = -15$ dBm $\quad P_\pi = -50$ dBm

$V = a n_1 k_0 \sqrt{2\Delta} = 27.35$

$M \approx \frac{V^2}{2} \approx 374$

b) Inter:

$\frac{\Delta t}{L} = |M|\Delta\lambda = 350$ ps/Km

Intra:

$\frac{\Delta t}{L} \approx \frac{n_1 \Delta}{c_0} \cdot 10^3 = 24.(6)$ ns/Km

c) $\Delta t = \Delta t_{inter} + \Delta t_{intra}$

$B = \frac{1}{2\Delta t} \Rightarrow BL = \frac{1}{2\left(\frac{10^3 n_1 \Delta}{c_0} + |M|\Delta\lambda\right)} = \frac{1}{2(0.35 + 24.6) \times 10^{-9}} \approx 20$ Mb · Km/s

$B = 1$ Mb/s $\rightarrow L = 20$ Km

$B = 34$ Mb/s $\rightarrow L = 588$ m

d) $A_{max} = -15 - (-50) = 35$ dB

$L_{max} = \frac{35}{2} = 17.5$ Km

O limite depende do ritmo de transmissão esperado

Para B < 1.14 Mb/s o limite é a atenuação

Para B > 1.14 Mb/s o limite é a dispersão

12. a) $n_1 = 1.45$ $\quad AN = \sqrt{n_1^2 - n_2^2} = 0.24$

$n_2 = 1.43$

$L = 20\,Km$ $\quad \Delta = \frac{AN^2}{2n_1^2} = 1.3698\,\%$

$\lambda_0 = 1.55\,\mu m$ $\quad \theta_a = \sin^{-1}(AN) = 13.89^0$

$\phi_{Lim} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = 80.47^0$

I b) $2a = 50\,\mu m \Leftrightarrow a = 25\,\mu m$ $\quad V = a\,K_0\,AN = a\,\frac{2\pi}{\lambda_0}\,AN = 24.322$

$M = 15\ pS/Km.nm$ $\quad \hookrightarrow V > 2.405 \Rightarrow$ Multimodal

$\Delta\lambda = 50\,nm$

c) Δt_{inter} :

$\Delta t \approx \frac{L}{c_0} n_1 \Delta = 1.324\,\mu S$

Δt_{intra} :

$\Delta t = L|M|\Delta\lambda = 15\,nS$

d) $\Delta t_{total} = \Delta t_{inter} + \Delta t_{intra}$

$B = \frac{1}{2\Delta t_{total}} = 374.375\,Kb/S$

II $2a = 4,9\,\mu m \Rightarrow a = 2.45\,\mu m$

e) $V = a K_0\,AN = 2.3836$

$\hookrightarrow V < 2.405 \Rightarrow$ Monomodal

f) Não há inter!

$\Delta t_{intra} = L|M|\Delta\lambda = 15\,nS$

$B = \frac{1}{2\Delta t} = 33.(3)\,Mb/S$

III $\Delta\lambda = 5\,nm$

g) $\Delta t_{intra} = L|M|\Delta\lambda = 1.5\,nS$

$B = \frac{1}{2\Delta t} = 333.(3)\,Mb/S$

f) I $\rightarrow$ Multimodal $\Rightarrow$ Mais distorção

Mais fácil de implementar

II $\rightarrow$ Monomodal ($\Delta\lambda$ grande) $\Rightarrow$ Menos distorção

Mais bit rate que I

Mais difícil e mais caro

III $\rightarrow$ Monomodal ($\Delta\lambda$ pequeno) $\Rightarrow$ Ainda menos distorção

Mais bit rate que I e II

Mais difícil e mais caro (fig é mono e Laser)