1. Onda plana

$f = 9.4\,GHz$

$\varepsilon_r = 2.25 \rightarrow \varepsilon = 2.25\,\varepsilon_0$

$\mu_r = 1 \rightarrow \mu = \mu_0$

$|\underline{H}| = 7\,mA/m$

a) $v_f = \frac{\omega}{K} = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = 2 \times 10^8\,m/s$

$K = \omega\sqrt{\varepsilon\mu}$

b) $\lambda = \frac{v_f}{f} = \frac{\omega}{K f} = \frac{v_f}{f} = 0.021\,m$

c) $K = \omega\sqrt{\varepsilon\mu} = \frac{\omega}{c} \cdot \sqrt{\varepsilon_r \mu_r} = 295.3\ m^{-1}$

d) $Z = Z_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} = Z_0 \sqrt{\frac{1}{\varepsilon_r}} = \frac{Z_0}{\sqrt{\varepsilon_r}} = 251.3\,\Omega = 80\pi\,\Omega$

e) $|\underline{E}| = |Z| \cdot |\underline{H}| = 1.76\ V/m$

2. Onda plana $P(4,-2,6)$

$f = 100\,MHz$ $\underline{\vec{E}} = 100\hat{x} - 70\hat{y}\ V/m$

a) Para $t=0$

$\underline{\vec{E}} = 100\hat{x} - 70\hat{y}$

$(100,-70,0) = (E_{0x}, E_{0y}, E_{0z})\, e^{j(\omega t - K\cdot r)}$

$100 = E_{0x}\, e^{-j\underbrace{(4K_x - 2K_y + 6K_z)}_{C>0}} \rightarrow E_{0x} = 100$

$-70 = E_{0y}\, e^{-j(0)} \rightarrow E_{0y} = -70$

Logo $\vec{E} = (100, -70, 0) = 100\hat{x} - 70\hat{y}$

b) $t = 1\,ms$

$\vec{E} = \vec{E}_0\, e^{j(\omega t - \overbrace{K\cdot r}^{\neq 0})} \rightarrow \vec{E} = Real\{(100,-70,0)\, e^{j(2\pi \cdot f \cdot 1ms)}\} = 80.9\hat{x} - 56.6\hat{y}$

c)

$\vec{r}_f = (3,5,8)$

$\vec{r}_i = (4,-2,6)$

$t = 2ms$

$\vec{m} = (0,0,1)$

$\vec{E} = Real\{\vec{E}_0\, e^{j(\omega t - (\vec{K}\cdot\vec{r}_f - \vec{K}\cdot\vec{r}_i))}\} = Real\{\vec{E}_0\, e^{j(\omega t - (8k_z - 6k_z))}\} =$

$= Real\{\vec{E}_0\, e^{j\omega\left(t - \frac{2}{c}\right)}\} =$

$\vec{K} = K\cdot\vec{m} = k_z \Rightarrow \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c}$

$= -97.8\hat{x} + 68.9\hat{y}\ V/m$

3.

$\vec{m} = \hat{z}$

a)

$t=0 \rightarrow Real(\vec{E_1}) = Real(\vec{E_2}) = 1\ V/m \rightarrow$ máximo

$\vec{E_1} = E_{1x}\, e^{\cdots}\hat{x} \rightarrow \vec{E_1} = e^{j(\omega t - r_z \cdot \frac{\omega}{c})} = e^{j\omega_1\left(t - \frac{r_{z1}}{c}\right)}\hat{x}$

$\vec{E_2} = E_{2x}\, e^{\cdots}\hat{x} \rightarrow \vec{E_2} = e^{j\omega_2\left(t - \frac{r_{z2}}{c}\right)}\hat{x}$

Mas como para $t=0$ temos r_z positivos

$t - \frac{r_{z1}}{c} = 2\pi K \rightarrow \frac{r_{z1}}{c}$ tende para $0 \rightarrow r_{z1}$ e $r_{z2} = 0$

Logo $\vec{E_1} = e^{j\omega_1 t}\hat{x}$

$\vec{E_2} = e^{j\omega_2 t}\hat{x}$

e para $Real\,\vec{E_1} = Real\,\vec{E_2} = 1$

temos $\omega_1 t = 2\pi K \mid K \in \mathbb{Z} \rightarrow t = T_1 K \rightarrow \frac{T_1}{T_2} = \frac{l}{K} \rightarrow \frac{l}{K} = \frac{93}{92}$

$\omega_2 t = 2\pi l \mid l \in \mathbb{Z} \quad t = T_2 l$

$l = 93$

$K = 92$

$t = \frac{K}{f_1} = \frac{l}{f_2} = 100\,\mu s$

b) Neste caso $\vec{E_1} = e^{j\omega_1\left(t - \frac{r_{z1}}{c}\right)}\hat{x} = e^{-j\frac{\omega_1 \cdot r_{z1}}{c}}\hat{x}$

$t=0$ $\vec{E_2} = e^{-j\frac{\omega_2 \cdot r_{z2}}{c}}\hat{x}$

$\vec{E}_{total} = 2\hat{x} \Rightarrow Real(\vec{E_1}) + Real(\vec{E_2}) = 2 \Rightarrow Real(\vec{E_1}) = Real(\vec{E_2}) = 1$

$\begin{cases} \frac{\omega_1 \cdot r_z}{c} = K2\pi \mid K \in \mathbb{Z} \\ \frac{\omega_2 \cdot r_z}{c} = l2\pi \mid l \in \mathbb{Z} \end{cases} \Rightarrow \begin{cases} r_z = \frac{cK}{f_1} \\ r_z = \frac{cl}{f_2} \end{cases} \Rightarrow \frac{K}{f_1} = \frac{l}{f_2} \Leftrightarrow \frac{K}{l} = \frac{f_1}{f_2} = \frac{92}{93}$

Menores valores: $K = 92$, $l = 93$

$r_z = \frac{cK}{f_1} = \frac{cl}{f_2} = 3 \times 10^4\,m = 30\,Km$

4.

$\vec{E} = Re\{(50\hat{x} - 120\hat{y})\, e^{j(10^9 t - \beta z)}\}$

$\varepsilon_r = 2.25$

$\mu_r = 1$ $\beta = ? = \frac{\omega}{c_0}\sqrt{\mu_r \varepsilon_r} = 50$ $\lambda = \frac{2\pi}{\beta} = 0.13m$

$\underline{H} = \frac{1}{Z}(\hat{m} \times \underline{E}) = \frac{1}{Z}(50\hat{y} + 120\hat{x})\, e^{j(\omega t - \beta z)}$

$Z = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}} = 80\pi$

$\hat{m} = (0,0,1)$

Logo $H(\pi, t) = (0.48\hat{x} + 0.2\hat{y})\cos(\omega t - \beta z)$

5. $\lambda_1 = 12\,cm \rightarrow \lambda_2 = 8\,cm$

$E_{0_1} = 60\,V/m$

$H_{0_2} = 0.1\,A/m$

μ_2 e ε_2 não reais

Assumindo que λ_1 é no ar

Logo $\lambda_1 = \frac{2\pi}{K} = \frac{2\pi}{\omega\sqrt{\varepsilon_0\mu_0}}$

$f = \frac{2\pi c}{2\pi\lambda_1} = \frac{c}{\lambda_1} = 2.5\,GHz$

$|Z| = \frac{60}{0.1} = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

$\lambda_2 f = \frac{1}{\sqrt{\mu_2\varepsilon_2}}$

$\Rightarrow$ $500 = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$

$\frac{\lambda_2 f}{c} = \frac{1}{\sqrt{\mu_r\varepsilon_r}}$

$\Leftrightarrow \begin{cases} 1.76 \approx \frac{\mu_r}{\varepsilon_r} \\ \mu_r\varepsilon_r = 2.25 \end{cases} \Leftrightarrow \begin{cases} 1.76\,\varepsilon_r = \mu_r \approx 1.99 \\ \varepsilon_r = \sqrt{\frac{2.25}{1.76}} \approx 1.13 \end{cases}$

Logo

$\mu = \mu_r\mu_0 \approx 1.99\,\mu_0$

$\varepsilon = \varepsilon_r\varepsilon_0 \approx 1.13\,\varepsilon_0$

$\sigma = 0$

6. $f = 1\,MHz$ a) $tan(\theta_p) = \frac{\sigma}{\omega\varepsilon} = 0.288$

$\varepsilon_r = 2.5$ b) e c) $\gamma = \omega\sqrt{\mu\varepsilon^*} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 0.0047 + j0.033$

$\mu_r = 1$

$\sigma = 4 \times 10^{-5}$ $\alpha = Re\{\gamma\} = 0.0047$

$\beta = Im\{\gamma\} = 0.033$

7. $|Z| = 200$

$f = 100\,MHz$

$\lambda = 1\,m \Rightarrow \beta = \frac{2\pi}{1} = 2\pi$

$\alpha = 2\,Np/m$

$\gamma = 2 + 2\pi j$ $2 + 2\pi j = j\omega\sqrt{\mu\varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)} \Leftrightarrow$

$\gamma = j\omega\sqrt{\mu\varepsilon^*} \Leftrightarrow 2 + 2\pi j = \sqrt{-\omega^2\mu\varepsilon + \mu\sigma\omega j} \Rightarrow$

$\varepsilon^* = \varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right) \Rightarrow (2 + 2\pi j)^2 = -\omega^2\mu\varepsilon + \mu\sigma\omega j \Leftrightarrow$

$\Leftrightarrow 4 - 4\pi^2 + 8\pi j = -\omega^2\mu\varepsilon + \mu\sigma\omega j$

$|Z|^2 = \left|\frac{\mu}{\varepsilon + \frac{\sigma}{j\omega}}\right| \Leftrightarrow |Z|^2 = \frac{\mu}{\sqrt{\varepsilon^2 + \left(\frac{\sigma}{\omega}\right)^2}}$

$\begin{cases} 4 - 4\pi^2 = -\omega^2\mu\varepsilon \\ 8\pi = \mu\sigma\omega \end{cases}$

Logo:

$\begin{cases} 4 - 4\pi^2 = -4\pi^2 f^2 \mu\varepsilon \\ 8\pi = 2\pi f\mu\sigma \\ |Z|^2 = \frac{\mu}{\sqrt{\varepsilon^2 + \left(\frac{\sigma}{\omega}\right)^2}} \end{cases} \Leftrightarrow \begin{cases} 1 - \frac{1}{\pi^2} = \varepsilon \Rightarrow \varepsilon^2 = \frac{1 - \frac{2}{\pi^2} + \frac{1}{\pi^4}}{f^4\mu^2} \\ \frac{4}{f\mu} = \sigma \Rightarrow \left(\frac{\sigma}{\omega}\right)^2 = \frac{4}{f^4\pi^2\mu^2} \end{cases}$

Logo $|Z|^2 = \frac{f^2\mu^2}{\underbrace{\sqrt{1 - \frac{2}{\pi^2} + \frac{1}{\pi^4} + \frac{4}{\pi^2}}}_{A}} \Leftrightarrow \mu = \sqrt{\frac{|Z|^2 \cdot A}{f^2}} = 2.044 \times 10^{-6}$

Como $\mu_r = \frac{\mu}{\mu_0}$ temos $\mu_r = 1.67$!

Para $\varepsilon = \frac{1 - \frac{1}{\pi^2}}{f^2\mu} \approx 42.8 \times 10^{-12}$ Para $\sigma = \frac{4}{f\mu} = 0.019$

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 4.84$

Logo $\mu_r = 1.67$

$\varepsilon_r = 4.84$

$\sigma = 0.019\ S/m$

8. $\omega = 2\,M\,Rad/s$

a) e b) $tan(\theta_p) = \frac{\sigma}{\omega\varepsilon} = 2.26$

$\underline{\vec{E}} = 20j\hat{x}\,[mV/m]$ em $z=0$

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = (28.6 + 43.9j) \times 10^{-3}$

$\varepsilon_r = 2.5$

$\mu_r = 10$

$\sigma = 10^{-4}$

$\alpha = Re\{\gamma\} = 0.0286$

$\beta = Im\{\gamma\} = 0.0439$

c) $\lambda = \frac{2\pi}{\beta} = 143.125\ m$

d) $v_f = \frac{\omega}{\beta} = 45.56 \times 10^6\,m/s$

e) $Z = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = 402 + j262\,j$

f) $\underline{\vec{E}} = E_0 e^{-\alpha z} e^{j(\omega t - \beta z + \theta)}$

$z=0 \Rightarrow 20j = E_0 e^{j\theta}$

$\theta = \frac{\pi}{2}$ e $E_0 = 20$

$E = Re\{20\, e^{-\alpha z}\, e^{j(\omega t - \beta z + \theta)}\} =$

$= 12.687\hat{x}\ mV/m$

9.

$f = 2.5\,GHz$

$\varepsilon^* = 30(1 - 0.3j)\varepsilon_0$

a) $\lambda = \frac{2\pi}{\beta} \simeq 21.67\,mm$

$\beta = Im\{\gamma\} = Im\{\sqrt{-\omega^2\mu\varepsilon(1 + \frac{\sigma}{j\omega\varepsilon})}\} = 289.927$

↳ $\varepsilon_R\varepsilon_0 = \varepsilon = 30\varepsilon_0$

$0.3 = \frac{\sigma}{\omega\varepsilon}$

b) $\delta = \frac{1}{\alpha} = \frac{1}{Re\{\gamma\varepsilon\}} = 0.0235m = 23.5\,mm$

c) $\delta_{PRATO} = \frac{1}{\alpha_{PRATO}} = \frac{1}{Re\{\gamma_{PRATO}\}} = \frac{1}{5\times10^{-3}} = 200\,m$ — 3 mm mão afetam !!

$\varepsilon^* = 1.1(1 - \frac{2\times10^{-4}}{1.1}j)\varepsilon_0$

↳ $\varepsilon_R\varepsilon_0 = \varepsilon = 1.1\varepsilon_0$

$\frac{2\times10^{-4}}{1.1} = \frac{\sigma}{\omega\varepsilon}$

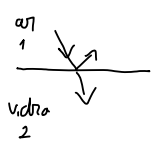
d) $\varepsilon = 80\varepsilon_0$

$\sigma = 10^{-3} \Rightarrow \frac{\sigma}{\omega\varepsilon} = \frac{10^{-3}}{2\pi f \cdot 80 \cdot \varepsilon_0}$

$\delta_{agua} = \frac{1}{Re\{\gamma_{agua}\}} = 47.45\,m$

10. $n = \sqrt{\varepsilon_R \mu_R}$

$\mu_R = 1$ $\Rightarrow$ $\varepsilon_R = 1.5^2 = 2.25$



$R = \frac{\sqrt{\frac{1}{2.25}} - 1}{\sqrt{\frac{1}{2.25}} + 1} = -0.2 \Rightarrow \frac{S_R}{S_{av}} = R^2 \Rightarrow 0.04 \rightarrow 4\%$

11. $|E_R| = R\,|E_i|$ $\quad Z_{ar} = Z_0$ $\quad Z_d = Z_0\frac{1}{2}$

$R = \frac{\frac{1}{2} - 1}{\frac{1}{2} + 1} = -\frac{1}{3} \rightarrow E_R = -33.3\,V/m$

$|E_T| = (1 - R)|E_i| \rightarrow E_T = 66.7\,V/m$

12. $\sin(\theta_T)\,n_{clad} = \sin(\theta_i)\,n_{core} \Rightarrow \theta_{lim} = \arccos\left(\frac{1.5}{1.5(1+0.01)}\right) \simeq \arccos(1 - 0.01) = 1.429$

$\theta_T = 90°$

$\phi = \frac{\pi}{2} - \theta_{lim} = 0.14 \rightarrow 8.1°$