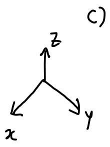
4. a) $\vec{E} = 10^{-2}\left[\sqrt{2}\,\hat{x} + (1+j)\,e^{j\frac{\pi}{4}}\,\hat{z}\right] e^{-jky}$ $\qquad \hat{m} = \hat{y}$

b) ↳

c) $E_x(t) = 10^{-2}\cdot\sqrt{2}\,\cos(\omega t + \phi)$



$E_z(t) = 10^{-2}\cdot\sqrt{2}\,\cos(\omega t + \phi + \frac{\pi}{2}) = -10^{-2}\sqrt{2}\,\sin(\omega t + \phi)$

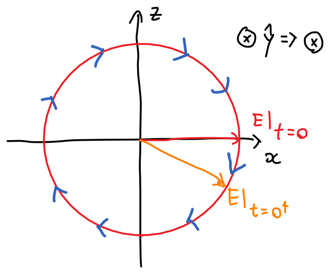
Eliminando o tempo: $\sqrt{E_x^2 + E_z^2} = E_0 \Rightarrow$ Circular

Sentido de rotação

$\omega t + \phi = 0 \qquad \omega t + \phi = 0^+$

$E_x = E_0 \qquad E_x < E_0$

$E_z = 0 \qquad E_z < 0 \Rightarrow$



$\otimes\hat{y} \Rightarrow \otimes\hat{m}$

$\circlearrowleft$ e $\otimes\hat{m}$
⇓
Polarização Circular Direita

5. a) b) c) $\vec{H} = \frac{10^{-3}}{120\pi}(\hat{x} - j\,\hat{z})\,e^{jky}$

$\hat{m} = -\hat{y} \quad \rightarrow \vec{E} = -Z_0(\hat{m}\times\vec{H}) =$

$= -Z_0\,\frac{10^{-3}}{120\pi}\underbrace{(\hat{m}\times\hat{x} - j\cdot\hat{m}\times\hat{z})}\,e^{jky}$

$\hat{m}\times\hat{z} = -\hat{y}\times\hat{x} = \hat{z} \Rightarrow \vec{E} =$
$\hat{y}\times\hat{z} = \hat{x}$

$E_z = -\overbrace{Z_0\frac{10^{-3}}{120\pi}}^{E_0}\cos(\omega t + \phi) = -E_0\cos(\omega t + \phi)$

$E_x = -\underbrace{Z_0\frac{10^{-3}}{120\pi}}_{E_0}\cos(\omega t + \phi + \frac{\pi}{2}) = E_0\sin(\omega t + \phi)$

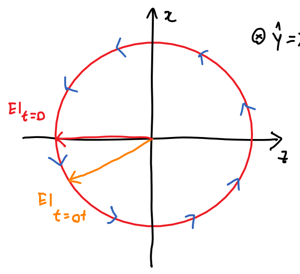
$\Rightarrow$ Polarização Circular

Sentido de rotação:

$\omega t + \phi = 0 \qquad \omega t + \phi = 0^t$

$E_z = -E_0 \qquad E_z > -E_0 \quad \rightarrow$

$E_x = 0 \qquad E_x < 0$



$\otimes\hat{y} \Rightarrow \odot\hat{m}$

$\circlearrowleft$ e $\odot\hat{m}$
⇓
Polarização Circular Direita

1. a) $\vec{H} = \frac{1}{120\pi}\,e^{-jkz}(\hat{x} - 2\hat{y}) \qquad Z = 120\pi \qquad \hat{x}\times\hat{z} = -\hat{y}$
$-2\hat{y}\times\hat{z} = -2\hat{x}$

$\vec{E} = Z\vec{H}\times\hat{m} = \frac{Z}{120\pi}\,e^{-jkz}(-\hat{y} - 2\hat{x}) = -e^{-jkz}(2\hat{x} + \hat{y})$

b) $\langle\vec{S}\rangle = \frac{1}{2}\mathrm{Re}\{\vec{E}\times\vec{H}^*\} = \frac{1}{2}\mathrm{Re}\left\{-e^{-jkz}(2\hat{x} + \hat{y})\times\frac{1}{120\pi}e^{+jkz}(\hat{x} - 2\hat{y})\right\} =$

$= \frac{1}{2}\mathrm{Re}\left\{-e^{-jkz}\cdot e^{+jkz}\frac{1}{120\pi}\cdot(-5)\cdot\hat{z}\right\} =$

$= \frac{5}{240\pi}\,\hat{z} = 6.63\times10^{-3}\,\hat{z}$

2. a) $\hat{m} = -\hat{z}$ $\qquad$ b)

$\vec{E}(z) = E_0\,e^{jkz}\,\hat{x} \qquad \vec{E}(z) = 4\times10^{-3}\,e^{jkz}\,\hat{x} \qquad \vec{E}(z,t) = 4\times10^{-3}\cos(\omega t + kz)\,\hat{x}$

$\vec{E}(0) = 4\times10^{-3} \qquad \vec{H}(z) = -\frac{4\times10^{-3}}{120\pi}\,e^{jkz}\,\hat{y} \qquad \vec{H}(z,t) = -\frac{4\times10^{-3}}{120\pi}\cos(\omega t + kz)\,\hat{y}$

$f = 300\,\mathrm{MHz} \qquad$ c) $\vec{S}(t) = -\frac{(4\times10^{-3})^2}{120\pi\cdot2}\left[\cos(2\omega t + 2kz) + 1\right]\hat{z}$

$\langle\vec{S}(t)\rangle = -\frac{(4\times10^{-3})^2}{120\pi\cdot2}\,\hat{z}$

3. $v_f = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\varepsilon\mu}} = c\cdot\frac{1}{\sqrt{9\cdot1}} = \frac{c}{3} = 1\times10^8$

$v_f = \lambda f \Leftrightarrow \lambda = \frac{v_f}{f} = \frac{1\times10^8}{1\times10^6} = 100\,\mathrm{m}$

$Z = \sqrt{\frac{\mu}{\varepsilon}} = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{120\pi}{3} = 40\pi\ \Omega$

6. $\vec{E} = Z\,\vec{H}\times\hat{m} = Z\,\frac{E_0}{Z}\,e^{-jky}\left[(1+j)\,\hat{x}\times\hat{y} + \sqrt{2}\,e^{j\frac{\pi}{4}}\,\hat{z}\times\hat{y}\right] =$

a) b) $= E_0(1+j)\,e^{-jky}\,\hat{z} - E_0(\sqrt{2}\,e^{j\frac{\pi}{4}})\,e^{-jky}\,\hat{x} =$

$= (1+j)\,E_0\,e^{-jky}(-\hat{x} + \hat{z})$

$e_1\times e_2 = \hat{y} \Rightarrow -\hat{x}\times\hat{z} = \hat{y}$

$P = \frac{E_0(1+j)}{E_0(\sqrt{2}\,e^{j\frac{\pi}{4}})} \Rightarrow \begin{cases}|P| = 1 & PL\\ \phi = 0\end{cases}$

7. a) $\vec{E}^i = 2\times10^{-3}\,e^{-j\beta z}\,\hat{y} \qquad Z = 120\pi\sqrt{\frac{1}{4}} = \frac{120\pi}{2} = 60\pi$

$\vec{H}^i = \hat{m}\times\frac{\vec{E}^i}{Z} = -\frac{10^{-3}}{30\pi}\,e^{-j\beta z}\,\hat{x}$

b) $R = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{120\pi - \frac{120\pi}{2}}{120\pi + \frac{120\pi}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$

$T = 1 + \frac{1}{3} = \frac{4}{3}$

c) $\vec{E}^R = \frac{1}{3}\,2\times10^{-3}\,e^{+j\beta z}\,\hat{y} \qquad \vec{E}^T = \frac{4}{3}\cdot2\times10^{-3}\,e^{-j\beta z}\,\hat{y}$

$\vec{H}^R = \frac{1}{3}\cdot\frac{10^{-3}}{30\pi}\,e^{j\beta z}\,\hat{x} \qquad \vec{H}^T = -\frac{4}{3}\,\frac{10^{-3}}{30\pi}\,e^{-j\beta z}\,\hat{x}$

d) $\vec{S}^i = \frac{1}{2}\mathrm{Re}\{\vec{E}^i\times\vec{H}^i\} = \frac{1}{2}\cdot\frac{1}{60\pi}\cdot|2\times10^{-3}|^2\,\hat{z} = \frac{4\times10^{-6}}{120\pi}\,\hat{z}$

$\vec{S}^R = -R^2\,\vec{S}^i = -\frac{1}{9}\cdot\frac{4\times10^{-6}}{120\pi}\,\hat{z}$

$\vec{S}^T = (1 - R^2)\,\vec{S}^i = \frac{8}{9}\,\frac{4\times10^{-6}}{120\pi}\,\hat{z}$

8.

$R^2\times100 = 5\%^R$

$(1 - R^2)\times100 = 5\%^T$

$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\frac{1}{9} - 1}{\frac{1}{9} + 1} = \frac{-\frac{8}{9}}{\frac{10}{9}} =$

$= -\frac{8}{10}$

$R^2\times100 = 8^2 = 64\%$ Refletida

$(1 - R^2)\times100 = 36\%$ Transmitida

9. a) $\varepsilon_r = 81 \qquad \theta_B = \cos^{-1}\left(\frac{1}{\sqrt{1 + \frac{1}{81}}}\right) = 6.34^\circ$

água | ar
1 | 2

$\theta_{lim} = \sin^{-1}\left(\frac{1}{9}\right) = 6.38^\circ$

b) $\varepsilon_r = 81 \qquad \theta_B = \cos^{-1}\left(\frac{1}{\sqrt{1 + 81}}\right) = 83.66^\circ$

ar | água
1 | 2

θ_{lim} = Não há!

c) $\varepsilon_r = 9 \qquad \theta_B = \cos^{-1}\left(\frac{1}{\sqrt{1 + \frac{1}{9}}}\right) = 18.43^\circ$

vidro | ar
1 | 2

$\theta_{lim} = \sin^{-1}\left(\frac{1}{3}\right) = 19.47^\circ$

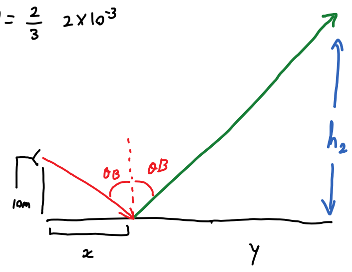
10. a) $R = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\frac{1}{2} - 1}{\frac{1}{2} + 1} = -\frac{1}{3}$

b) $SWR = \frac{1 + |R|}{1 - |R|} = 2$

c) $SWR = \frac{|E|_{max}}{|E|_{min}} \Rightarrow \quad E_{max} = E_0(1 + |R|) = \frac{4}{3}\,2\times10^{-3}$

$E_{min} = E_0(1 - |R|) = \frac{2}{3}\,2\times10^{-3}$

11. $R_{||} = 0 \Rightarrow$ Pede o ângulo de Brewster



$\theta_B = \cos^{-1}\left(\frac{1}{\sqrt{1 + 81}}\right) = 83.66^\circ$

$\tan(\theta_B) = \frac{x}{10} \Leftrightarrow x = 90$

$y = d - x = 10\,\mathrm{km} - 90\,\mathrm{m} = 9910$

$\tan(\theta_B) = \frac{y}{h_2} \Leftrightarrow h_2 = \frac{9910}{9} = 1101\,\mathrm{m} \rightarrow 1.10\,\mathrm{Km}$

12.

a) $Z_2 = \sqrt{\frac{\mu}{\varepsilon_c}} = Z_0\sqrt{\frac{1}{\varepsilon_r\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)}} \qquad \varepsilon_c = \varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)$

$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} = -0.99 + 0.01j$

c) $|\vec{S}^i_{av}| = \left|\frac{1}{2}\mathrm{Re}\left\{\frac{1}{Z}\right\}|\vec{E}^i|^2\right| = 1.32\,\mathrm{mW/m^2}$

$\%P_R = |R|^2 = 0.98$

$\%P_T = 1 - |R|^2 = 0.02$

$|\vec{S}^R_{av}| = |\vec{S}^i_{av}|\cdot0.98 = 1.29\,\mathrm{mW/m^2}$

b) $SWR = \frac{1 + |R|}{1 - |R|} = 189.3$

se $|R| = 0.99 \rightarrow SWR = 199$

d) $|E^t_0| = |(1 + R)\,E^i_0| = 14.8\,\mu\mathrm{V/m}$

e) $|\vec{S}^t_{av}| = 0.02\,|\vec{S}^i_{av}| = 26.5\,\mu\mathrm{W/m^2}$

ou

$|\vec{S}^t_{av}| = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{Z}\right\}|E^t_0|^2 = 24.7\,\mu\mathrm{W/m^2}$

f) $\delta = \frac{1}{\alpha}$

temos $\frac{\sigma}{\omega\varepsilon} = 122 \gg 1$!

Bom condutor

$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 0.5\,\mathrm{m}$

g) $\beta = \frac{2\pi}{\lambda} \Leftrightarrow \frac{1}{\delta} = \frac{2\pi}{\lambda}$

$\lambda = 2\pi\delta \Rightarrow d = 20\cdot2\pi\delta = 63.25\,\mathrm{m}$

i) $\frac{V_g}{c} = \frac{6.32}{3\times10^2} = 0.02$

h)

$t = \frac{100}{v_g} = 15.8\,\mu\mathrm{s}$

$v_g = \frac{\partial\omega}{\partial\beta} = \frac{4\beta}{\mu\sigma} \approx 6.32\times10^6\,\mathrm{m/s}$

$\beta = \sqrt{\frac{\omega\mu\sigma}{2}} \Leftrightarrow \frac{\beta^2\,2}{\mu\sigma} = \omega$

13\. a)

LCP $\hat{z} \rightarrow E_1 = 1e^{j\psi}$

$\hat{z} \times \hat{y} = -\hat{x}$ $\hat{y} \rightarrow E_2 = 1$

$P = \frac{E_2}{E_1} = 1e^{-j\psi}$

LCP $\rightarrow \psi = -\frac{\pi}{2}$

b) $\underline{\vec{H}} = \hat{n} \times \frac{\underline{\vec{E}}}{Z_0}$

$\underline{\vec{H}}^i = \frac{E_0}{Z_0}(-\hat{z} + \hat{y}e^{-j\frac{\pi}{2}})e^{jkx}$

c) $\underline{\vec{E}}^{ref} = R\,\underline{\vec{E}}^i$

$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \approx -0.8$

$\underline{\vec{E}}^{ref} = -0.8\,(\hat{y} + e^{j\frac{\pi}{2}}\hat{z})\,E_0 e^{jkx}$

$\underline{\vec{H}}^{ref} = -0.8\,\frac{E_0}{Z_0}(-\hat{z} + \hat{y}e^{j\frac{\pi}{2}})e^{jkx}$

d) É aproximadamente circular esquerda

e) $\underline{\vec{E}}^t = T\,\underline{\vec{E}}^i = (1+R)\,\underline{\vec{E}}^i \approx 0.2\,\underline{\vec{E}}^i$

$\underline{\vec{E}}^{ref} = 0.2\,(\hat{y} + e^{j\frac{\pi}{2}}\hat{z})\,E_0 e^{jkx}$

$\underline{\vec{H}}^{ref} = 0.2\,\frac{E_0}{Z_0}(-\hat{z} + \hat{y}e^{j\frac{\pi}{2}})e^{jkx}$

f) É aproximadamente circular esquerda

g)

$P_r\% = R^2 = 0.64 \rightarrow 64\%$ $\quad P_T = 1 - R^2 = 0.36 \rightarrow 36\%$

14\. a) $\theta_i = 45°$ $\rightarrow$ $\theta_t = \arcsin\left(\frac{n_1}{n_2}\sin(\theta_i)\right)$

Angulo com o plano $= 45° \rightarrow \hat{e}_\perp = \hat{e}_\parallel \rightarrow |\hat{e}_\perp + \hat{e}_\parallel| = 1 \Rightarrow \frac{(\hat{e}_\perp + \hat{e}_\parallel)}{\sqrt{2}}$

$$R_\perp = \frac{\frac{1}{\sqrt{2,7}}\cos(45°) - \cos(\theta_t)}{\frac{1}{\sqrt{2,7}}\cos(45°) + \cos(\theta_t)} = -0.36 \qquad R_\parallel = \frac{\cos(45°) - \frac{1}{\sqrt{2,7}}\cos(\theta_t)}{\cos(45°) + \frac{1}{\sqrt{2,7}}\cos(\theta_t)} = 0.13$$

$$\underline{\vec{E}_i} = (\hat{e}_\perp + \hat{e}_\parallel^i)\frac{E_0}{\sqrt{2}}e^{-jk\vec{r}\cdot\hat{n}_i} \rightarrow \underline{\vec{E}_r} = (R_\parallel \hat{e}_\parallel^r + R_\perp \hat{e}_\perp)\frac{E_0}{\sqrt{2}}e^{-jk\vec{r}\cdot\hat{n}_r}$$

$\theta_i = \theta_B$

$$R_\parallel = 0 \quad R_\perp = -0.45 \rightarrow \underline{\vec{E}_i} = (\hat{e}_\perp + \hat{e}_\parallel^i)\frac{E_0}{\sqrt{2}}e^{-jk\vec{r}\cdot\hat{n}_i} \rightarrow \underline{\vec{E}_r} = (R_\perp \hat{e}_\perp)\frac{E_0}{\sqrt{2}}e^{-jk\vec{r}\cdot\hat{n}_r}$$

b)

$\theta_i = 45°$

$$\frac{|\vec{S}|_{ref}}{|\vec{S}|_{inc}} = \frac{\frac{1}{2}Re\{\frac{1}{Z}\}|\underline{\vec{E}_r}|^2}{\frac{1}{2}Re\{\frac{1}{Z}\}|\underline{\vec{E}_i}|^2} = \frac{|\underline{\vec{E}_r}|^2}{|\underline{\vec{E}_i}|^2} = \frac{(R_\parallel^2 + R_\perp^2)\left(\frac{E_0}{\sqrt{2}}\right)^2}{(1^2+1^2)\left(\frac{E_0}{\sqrt{2}}\right)^2} = \frac{R_\parallel^2 + R_\perp^2}{2} = 7.325\%$$

$\theta_i = \theta_B$

$$\frac{|\vec{S}|_{ref}}{|\vec{S}|_{inc}} = \frac{|\underline{\vec{E}_r}|^2}{|\underline{\vec{E}_i}|^2} = \frac{R_\perp^2}{2} = 10.125\%$$

15\. PCR

$\sigma = 0,\ \mu_r = 1,\ \varepsilon_r = 5$

a) $p_i = \frac{E_\perp^i}{E_\parallel^i} = 1e^{-j\frac{\pi}{2}}$

$p_r = \frac{R_\perp}{R_\parallel}e^{-j\frac{\pi}{2}} = -2e^{-j\frac{\pi}{2}} = 2e^{j\frac{\pi}{2}}$ elíptica para a esquerda

$$R_\perp = \frac{Z_2\cos(\theta_i) - Z_1\cos(\theta_t)}{Z_2\cos(\theta_i) + Z_1\cos(\theta_t)} = \frac{\sqrt{\frac{1}{5}}\cdot\frac{1}{\sqrt{2}} - \sqrt{1-\frac{1}{10}}}{\sqrt{\frac{1}{5}}\cdot\frac{1}{\sqrt{2}} + \sqrt{1-\frac{1}{10}}} = \frac{\frac{1}{\sqrt{10}} - \frac{3}{\sqrt{10}}}{\frac{4}{\sqrt{10}}} = -\frac{1}{2}$$

$$R_\parallel = \frac{Z_1\cos(\theta_i) - Z_2\cos(\theta_t)}{Z_1\cos(\theta_i) + Z_2\cos(\theta_t)} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\sqrt{1-\frac{1}{10}}}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{5}}\sqrt{1-\frac{1}{10}}} = \frac{1}{4}$$

$\sin(\theta_t)\sqrt{5} = \sin(\theta_i)$

$\sin(\theta_t) = \frac{1}{\sqrt{10}} \Rightarrow \cos(\theta_t) = \sqrt{1-\frac{1}{10}}$

$Z_2 = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$

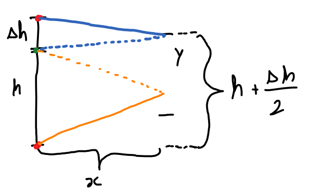
b)

$$\frac{|\vec{S}_r|}{|\vec{S}_i|} = \frac{|\vec{E}_r|^2}{|\vec{E}_i|^2} = \frac{R_\perp^2 + R_\parallel^2}{(1+1)} = 15.625\%$$

$|\vec{E}_r|^2 = (|R_\perp|^2 + |R_\parallel|^2)\ E^2$

$|\vec{E}_i|^2 = (|1|^2 + |e^{j\frac{\pi}{2}}|^2)\ E^2$

16\.



• - olhos
• - extremos

pela primeira lei de Snell! $\boxed{\theta_i = \theta_r}$

• - Reflexão topo cabeça → Para ver total temos de ter $\frac{\Delta h}{2}$ para cima dos olhos

• - Reflexão pés → Para ver total no mínimo precisamos de $\frac{h}{2}$ para baixo dos olhos

Logo $Y = \frac{\Delta h}{2} + \frac{h}{2}$

17\. ar - prisma | $\theta_L = \arcsin\left(\frac{n_2}{n_1}\right)$ | prisma - ar

$R_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$

$\theta_L = 41.8°$

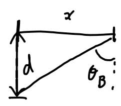
$\theta_{dentro} = 45°$

há reflexão total

$R_3 = \frac{Z_1 - Z_2}{Z_2 + Z_1}$

$(1 - R_1^2)\ (1 - R_2^2) = 92.16\%$

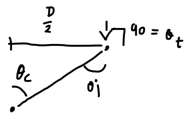
18\.



$\tan(\theta_B) = \sqrt{\frac{1}{81}} = \frac{1}{9}$

$\tan(\theta_B) = \frac{x}{d} \Leftrightarrow x_{max} = \frac{1}{9}d = 0.11\,d$ //

19\.



$\sin(\theta_t)\,1 = \sin(\theta_i)\,9$

$\frac{1}{9} = \sin(\theta_i)$

$\hookrightarrow \theta_i = 6.379°$