1. $V_p = \frac{1}{\sqrt{LC}}$ $\qquad \frac{V_p}{C_0} = \frac{1}{3\times10^8\sqrt{LC}}$

$L = 150\times10^{-9}$ H/m

$\rightarrow C_1 = 10\times10^{-9}$ F/m $\Rightarrow \frac{V_{P1}}{C} = 0.086$

$\rightarrow C_2 = 100\times10^{-9}$ F/m $\Rightarrow \frac{V_{P2}}{C} = 0.027$

$\rightarrow C_3 = 1\times10^{-6}$ F/m $\Rightarrow \frac{V_{P3}}{C} = 0.0086$

2. $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{R+j\omega L}\cdot\sqrt{G+j\omega C} =$

$= \sqrt{j\omega L\left(\frac{R}{j\omega L}+1\right)}\cdot\sqrt{j\omega C\left(\frac{G}{j\omega C}+1\right)} \simeq$

$\simeq \sqrt{j\omega L}\left(1+\frac{R}{2j\omega L}\right)\sqrt{j\omega C}\left(1+\frac{G}{2j\omega C}\right) =$

$= \sqrt{-\omega^2 LC}\left(1+\frac{R}{2j\omega L}+\frac{G}{2j\omega C}+\underbrace{\frac{GR}{-4\omega^2 CL}}_{<<\text{ Desprezável}}\right) \simeq$

$\simeq j\omega\sqrt{LC}\left(1+\frac{R}{2j\omega L}+\frac{G}{2j\omega C}\right) =$

$= \underbrace{\frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}}}_{\alpha} + j\underbrace{\omega\sqrt{LC}}_{\beta}$

Nota: $G <<$, $R <<$

$\sqrt{1+x} \simeq 1+\frac{x}{2}$, $x << 1$

3. $\frac{G}{C} = \frac{R}{L}$

a) $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L\left(\frac{R}{L}+j\omega\right)}{C\left(\frac{G}{C}+j\omega\right)}} = \sqrt{\frac{L}{C}} \rightarrow$ é real

b) As distorções acontecem quando a velocidade de fase é diferente para diferentes frequências, ou seja, as frequências diferentes sobrepõem-se deformando assim o sinal.
Para isto não acontecer a Vp (velocidade de fase) não pode depender da frequência ou seja tem de ser constante.

$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} \rightarrow$ constante!

$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{LC\left(\frac{R}{L}+j\omega\right)\left(\frac{G}{C}+j\omega\right)} =$

$= \sqrt{LC}\left(\frac{R}{L}+j\omega\right) = \sqrt{LC}\,\frac{R}{L} + j\omega\sqrt{LC}$

$\alpha = R\sqrt{\frac{C}{L}} \qquad \beta = \omega\sqrt{LC}$

4. $V(t) = 5\cos(\omega t+\beta x) + 10\sin(\omega t-\beta x)$

$Z_0 = 50\ \Omega \qquad x_L = \frac{\lambda}{4}$

a) $I(t) = \frac{V_i(t)}{Z_0} - \frac{V_R(t)}{Z_0} = -0{,}1\cos(\omega t+\beta x) + 0{,}2\sin(\omega t-\beta x)$

b) $K_V = \frac{V_R(t)}{V_i(t)} = \frac{5\cos(\omega t+\beta x)}{10\sin(\omega t-\beta x)} = \frac{5e^{j(\omega t+\beta x)}}{10e^{j(\omega t-\beta x-\frac{\pi}{2})}} = \frac{1}{2}e^{j\left(2\beta x+\frac{\pi}{2}\right)}$

$x = x_L = \frac{\lambda}{4}$, $\beta = \frac{2\pi}{\lambda}$

$K_V(x_L) = \frac{1}{2}e^{j\left(2\cdot\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}+\frac{\pi}{2}\right)} = \frac{1}{2}e^{j\frac{3\pi}{2}} = -\frac{1}{2}j$

c) $Z(x) = Z_0\frac{1+K_V}{1-K_V} \rightarrow Z\left(\frac{\lambda}{4}\right) = 50\cdot\frac{1-\frac{1}{2}j}{1+\frac{1}{2}j} = 30-40j\ \Omega$

5. a)

$|Z_{in}| = \frac{|V_{in}|}{|I_{in}|} = \frac{1}{15\times10^{-3}} = \frac{200}{3}\ \Omega$

Como $l = \frac{\lambda}{4}$ e não há perdas e Z_L real $\Rightarrow Z_{in} = |Z_{in}|$

$Z_{in} = \frac{Z_0^2}{Z_L} \Leftrightarrow Z_0 = \sqrt{Z_L\cdot Z_{in}} = 70\ \Omega$

b) $\angle Z_{in} = 0°$

6. $l = 1.5\lambda$

$Z_0 = 50\Omega$

$Z_L = 60\Omega$

$V_L = 20e^{j40°}$ V

a) $\begin{cases}\bar V_L = \bar V^+ + \bar V^- \\ K_V = \frac{\bar V^-}{\bar V^+}\end{cases} \Rightarrow \begin{cases}\bar V_L = \bar V^+(1+K_V) \Rightarrow \bar V^+ = \frac{\bar V_L}{(1+K_V)} \\ \bar V_L = \bar V^-\left(1+\frac{1}{K_V}\right) \Rightarrow \bar V^- = \frac{\bar V_L}{1+\frac{1}{K_V}}\end{cases}$ $\qquad K_V = \frac{60-50}{60+50} = \frac{1}{11}$

$P = \frac{1}{2}\frac{|V^+|^2}{Z_0} - \frac{1}{2}\frac{|V^-|^2}{Z_0} = 3.33$ W

ou

$P = \frac{1}{2}\mathrm{Re}\left\{\bar V_L\cdot\bar I_L^*\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{20^{j40°}\cdot 20^{-j40°}}{60}\right\} = \frac{1}{3}\text{ W} = 3.33\text{ W}$

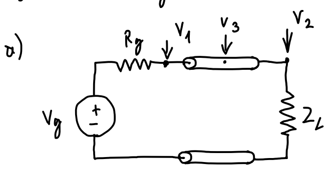
b) $V_{min} = |V^+|(1-K_V) = \left|\frac{\bar V_L}{1+K_V}\right|(1-K_V) = 16.67$ V

c) $I_{max} = \frac{|V^+|}{Z_0}(1+|K_V|) = 0.4$ A

7. $l = 0.25\lambda \quad V_g = 90$ V $\quad Z_L = 500\Omega$

$Z_0 = 300\Omega \quad R_g = 100\Omega$

a)



$V_1 = V_g - I_1R_g$

$Z_1 = Z_{\frac{\lambda}{4}} = \frac{V_1}{I_1} = \frac{Z_0^2}{Z_L}$

$I_1 = \frac{V_g}{\frac{Z_0^2}{Z_L}+R_g} = \frac{9}{28}$

$V_1 = V_g - \frac{V_g}{\frac{Z_0^2}{Z_L}+R_g}R_g = \frac{405}{7}$

$P = \frac{1}{2}\mathrm{Re}\{V_1I_1^*\} = \frac{1}{2}\cdot\frac{405}{7}\cdot\frac{9}{28} = 9.3$ W

$2\gamma z = 2\cdot\frac{2\pi}{\lambda}\frac{\lambda}{4}j = \pi j$

b) $K_{V_0} = \frac{Z_L-Z_0}{Z_L+Z_0} = 0.25 = \frac{\bar V_{R0}}{\bar V_{i0}}$

$V_1 = \bar V_{R0}e^{-\gamma d} + \bar V_{i0}e^{\gamma d} = \bar V_{R0}e^{-j\frac{\pi}{2}} + \bar V_{i0}e^{j\frac{\pi}{2}} = -\bar V_{R0}j + \bar V_{i0}j$

$\frac{403}{7} = -0.25\bar V_{i0}j + \bar V_{i0}j \rightarrow \bar V_{i0} = \frac{405}{7(-0.25j+j)}$

$V_2 = V_{i0} + 0.25V_{i0} = -96.43j$ $\qquad$ (Podemos usar matrizes)

c) $Z_L = \frac{V_2}{I_2} \qquad \frac{1}{2}\mathrm{Re}\left\{\frac{V_2\cdot V_2^*}{Z_L}\right\} = 9.3$ W

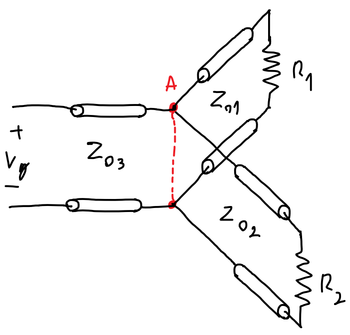
Numa linha sem perdas, a potência fornecida é a recebida

d) $V_3 = V\left(\frac{\lambda}{8}\right) = \bar V_{R0}e^{-\gamma\frac{\lambda}{8}} + \bar V_{i0}e^{\gamma\frac{\lambda}{8}} = 0.25\bar V_{i0}e^{-j\frac{\pi}{4}} + \bar V_{i0}e^{j\frac{\pi}{4}} =$

$= 40.9 - 68.2j = 79.5e^{-j59°}$

8. $P_1 = 2P_2 \qquad R_1 = 300\Omega \quad Z_{03} = 100\Omega$

$K_V = 0 \Rightarrow$ Carga adaptada $\qquad R_2 = 200\Omega$



$Z_1(A) = Z_1\left(\frac{\lambda_1}{4}\right) = \frac{Z_{01}^2}{R_1} \qquad Z_2(A) = Z_2\left(\frac{\lambda_2}{4}\right) = \frac{Z_{02}^2}{R_2}$

Para $K_V = 0$ $\quad Z_{03} = Z_1(A)\,//\,Z_2(A) =$

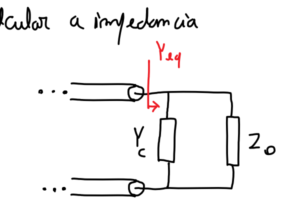
$= \frac{Z_{01}^2\cdot Z_{02}^2}{R_1Z_{02}^2 + R_2Z_{01}^2}$

$P_2 = \frac{1}{2}\cdot\frac{V_A^2}{Z_2(A)} = \frac{1}{2}\cdot\frac{R_2}{Z_{02}^2}V_A^2 \qquad P_1 = \frac{1}{2}\cdot\frac{R_1}{Z_{01}^2}V_A^2$

$P_1 = 2P_2 \Leftrightarrow 2\frac{R_2}{Z_{02}^2} = \frac{R_1}{Z_{01}^2} \Leftrightarrow \frac{2R_2}{R_1}Z_{01}^2 = Z_{02}^2$

$\begin{cases}\frac{2R_2}{R_1}Z_{01}^2 = Z_{02}^2 \\ \frac{Z_{01}^2\cdot Z_{02}^2}{R_1Z_{02}^2+R_2Z_{01}^2} = 100\end{cases}$ $\qquad \begin{matrix}Z_{01}^2 = 45000 \rightarrow Z_{01} = 212\ \Omega \\ Z_{02}^2 = 60000 \rightarrow Z_{02} = 245\ \Omega\end{matrix}$

9) Calcular a impedancia



$$Y_0 = \frac{1}{Z_0}$$

$$Y_{eq} = Y_c + Y_0$$

$$K_V = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{\frac{1}{Z_0} - \frac{1}{Z_L}}{\frac{1}{Z_0} + \frac{1}{Z_L}} = \frac{Y_0 - Y_{eq}}{Y_0 + Y_{eq}} = \frac{-Y_c}{2Y_0 + Y_c}$$

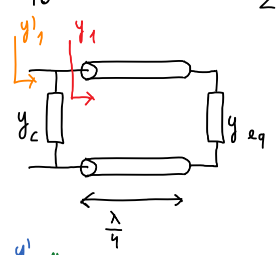
$$Y_c = \frac{2K_V Y_0}{-1 - K_V} \approx 4 \times 10^{-5} e^{j\frac{\pi}{2}}$$ (Usar Y é mais prático)

a) $\lambda = \frac{0.75}{f} c = 45\,cm$

$d = 11.25\,cm = \frac{\lambda}{4}$

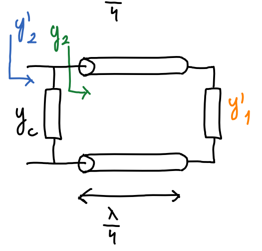
$$y_{eq} = \frac{Y_{eq}}{Y_0} = 1 + y_c$$

$$Z_1 = \frac{Z_0^2}{Z_{eq}} \Rightarrow Y_1 = \frac{Y_0^2}{Y_{eq}} \Rightarrow y_1 = \frac{1}{y_{eq}}$$



$$y_1 = \frac{1}{y_{eq}} = \frac{1}{1 + y_c} \simeq 1 - y_c$$ (Série de Taylor)

$$y'_1 = y_c + y_1 = 1$$



$$y_2 = \frac{1}{y'_1} = 1 \Rightarrow y_2 = y_L \;!!$$

$$y'_2 = y_2 + y_c = 1 + y_c \Rightarrow y'_2 = y_{eq} \;!!$$

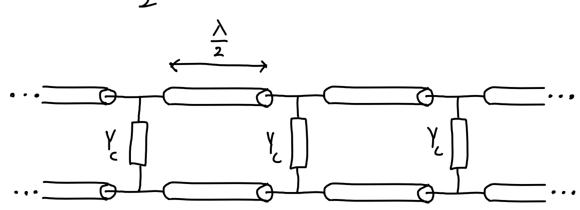
$$N = \left\lfloor \frac{50\,m}{11.25\,cm} \right\rfloor = 444$$

2 corrugações anulam-se e como $\frac{444}{2}$ é inteiro

$\hookrightarrow Z_{in} = Z_L \Rightarrow K_V = 0$

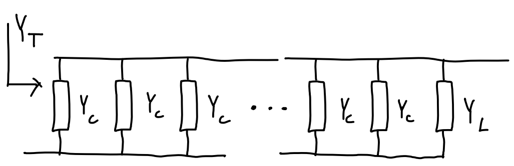
b) $\lambda = \frac{0.75}{f} c = 22.5\,cm$

$d = 11.25 = \frac{\lambda}{2}$



Transformador $\frac{\lambda}{2}$: $Z(\frac{\lambda}{2}) = Z_L$

Logo:



$$Y_T = N Y_c + Y_0 \qquad N = \left\lfloor \frac{50\,m}{11.25\,cm} \right\rfloor = 444$$

$$K_V = \frac{Y_0 - Y_T}{Y_0 + Y_T} = \frac{-N Y_c}{2Y_0 + N Y_c} = 0.41 e^{-j114^\circ}$$

c) $\lambda = \frac{0.75}{f} c = 11.25\,cm$

$d = 11.25 = \lambda \qquad \beta\lambda = 2\pi$

$$Z(\lambda) = Z_0 \frac{Z_L + jZ_0 \tan(\beta\lambda)}{Z_0 + jZ_L \tan(\beta\lambda)} = Z_L$$

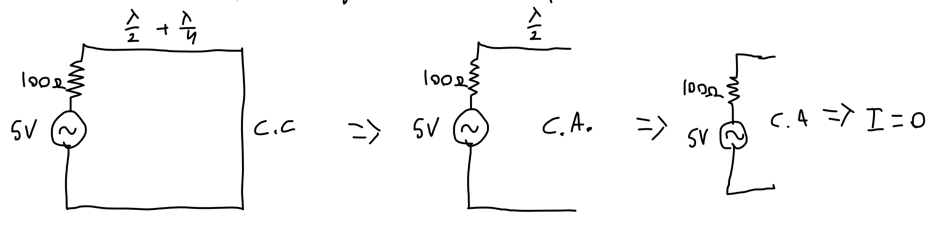
É a mesma situação que b $\Rightarrow K_V = 0.41 e^{-j114^\circ}$

10) c.c. visto de Z_{L3} (transformador $\frac{\lambda}{2}$)

$$Z_L = 0 \xrightarrow[\frac{\lambda}{2}]{} Z'_L = 0$$

$$Z'_{L3} = Z_{L3} // Z'_L = \left(\frac{1}{50j} + \frac{1}{0}\right)^{-1} = 0$$

Ou seja como até Z_{L1} são transformadores $\frac{\lambda}{2}$ temos que $Z_L(\frac{\lambda}{2} + \frac{\lambda}{4}) = 0$



temos então um transformador $\frac{\lambda}{4}$ seguido por um $\frac{\lambda}{2}$

Logo $Z(\frac{\lambda}{2}) = \frac{Z_0^2}{c.c} = \infty \Rightarrow$ transformador $\frac{\lambda}{4}$ leva a um circuito aberto

$Z(\text{gerador}) = \infty \Rightarrow$ transformador $\frac{\lambda}{2}$