

1.

$$E = 100 \frac{I}{\pi}$$

$$P = \iint_{\Sigma} dS \cdot \frac{1}{2} \frac{|E|^2}{Z_0} \cdot \hat{n} = 4\pi R^2 \cdot \frac{1}{2} \frac{100^2 |I|^2}{R^2 \cdot Z_0} \cdot \hat{n} =$$

$$= \frac{1}{2} |I|^2 \cdot \frac{4\pi}{120\pi} \cdot 100^2 \cdot \hat{n}$$

$$R_{\pi} = \frac{4\pi}{120\pi} \cdot 100^2 = 333. (3) \Omega$$

2.

$$\eta = 0.9 \quad \vec{H} = H_0 (\hat{\theta} - j\hat{\phi}) \sin(\theta) \frac{e^{-jk_0 r}}{k_0 r}$$

$$H_0 = 0.01 A/m \quad f = 3 \text{ GHz}$$

$$G_P = 4\pi \frac{U_{\max}}{P_{\text{im}}} = \eta D =$$

$$= \eta 4\pi \cdot \frac{U_{\max}}{P_{\pi}}$$

$$U = \pi^2 \cdot \vec{S}_{av} =$$

$$= \pi^2 \cdot \frac{1}{2} Z_0 |\vec{H}|^2 \hat{n} =$$

$$= \frac{Z_0 H_0^2}{k_0^2} \cdot \sin^2(\theta) \cdot \hat{n} \quad \left\{ \begin{array}{l} U_{\max} = \frac{Z_0 H_0^2}{k_0^2} \\ P_{\pi} = \int_0^{2\pi} \int_0^{\pi} \frac{Z_0 H_0^2}{k_0^2} \cdot \sin^3(\theta) \cdot d\theta d\varphi = \frac{Z_0 H_0^2}{k_0^2} \cdot 2\pi \cdot \frac{4}{3} \end{array} \right.$$

$$G_P = \eta \cdot 4\pi \cdot \frac{U_{\max}}{P_{\pi}} = \eta \cdot \frac{4\pi}{4\pi \cdot \frac{2}{3}} = \eta \cdot \frac{3}{2} = 1.35$$

3. I a)

$$R_P = \frac{\lambda}{50} \cdot 5 \text{ m}\Omega/\text{m} \quad \left| \rightarrow R_P = 2 \Omega \right.$$

$$\lambda = \frac{c}{f}$$

$$\eta = \frac{R_{\pi}}{R_{\pi} + R_P} = 0.136 = 13.6\%$$

$$R_{\pi} = 80 \pi^2 \left(\frac{\lambda}{\lambda} \right)^2 = 0.316 \Omega$$

$$b) E = 1 \text{ mV/m} \quad |E| = \left| j \frac{Z_0 I_0 L}{2\lambda} \cdot \frac{e^{-jk_0 r}}{\pi} \sin(\theta) \right|$$

$$R = 200 \text{ km}$$

$$\downarrow$$

$$|E| = \frac{Z_0 L}{2\lambda R} |I_0| \Leftrightarrow \frac{2\lambda |E|}{Z_0 L} = |I_0|$$

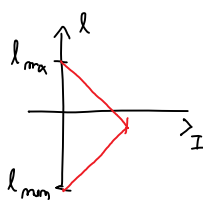
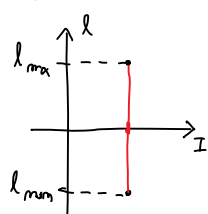
$$P_{\text{im}} = \frac{1}{2} R_a \cdot |I_0|^2 = 2 R_a \cdot \frac{\lambda^2 |E|^2 \pi^2}{Z_0^2 L^2} = 2 R_a \left(\frac{50 |E| \pi}{Z_0} \right)^2 = 3.26 \text{ kW}$$

$$P_{\pi} = \oint d^2 S_{av} d\Omega = d^2 \cdot \frac{1}{2} \cdot \frac{|E|^2}{Z_0} \int_0^{2\pi} \int_0^{\pi} \sin^3(\theta) d\theta d\varphi = d^2 \cdot \frac{4\pi}{3} \cdot \frac{|E|^2}{Z_0} = 444. (4) \text{ W}$$

$$P_{\text{im}} = \frac{P_{\pi}}{\eta} = 3.26 \text{ kW}$$

II c) DEH

DEC



d) Distribuição

$$R_{\pi H} = R_{P_H} + R_{\pi H}$$

$$R_{\pi C} = \frac{1}{3} R_{P_H} + \frac{1}{4} R_{\pi H}$$

$$R_{\pi C} = \frac{1}{4} R_{\pi H} \quad R_{P_C} = \frac{1}{3} R_{P_H}$$

$$\eta = 13.6\%$$

$$\left. \begin{array}{l} R_P = \frac{1}{3} R_{P_{DEH}} \\ R_{\pi} = \frac{1}{4} R_{\pi_{DEH}} \end{array} \right\} \eta = \frac{R_{\pi}}{R_{\pi} + R_P} = 10.54\%$$

l) + Capacitância $\Rightarrow$ - Facilidade na adaptação!

Pode ter bastante poder

4. $L = 3 \text{ cm}$

a) $\theta = 45^\circ$

$f = 100 \text{ MHz}$

$\pi_1 = 4 \text{ cm}$

$I_0 = 10 \text{ A}$

$\frac{2\pi R_1}{\lambda} = 0.08 \ll 1$ Comp. próxima

$$\vec{E} = \begin{cases} \vec{E}_r = \frac{Z_0 I_0 L}{2\pi \pi} \cdot \left(1 + \frac{1}{jk\pi}\right) \cdot \frac{e^{-jk\pi}}{\pi} \cos(\theta) = 95.29 e^{-j90.01^\circ} \text{ kV/m} \approx -95.29 j \text{ kV/m} \\ \vec{E}_\theta = j \frac{Z_0 k I_0 L}{4\pi} \left(1 + \frac{1}{jk\pi} + \frac{1}{(jk\pi)^2}\right) \cdot \frac{e^{-jk\pi}}{\pi} \sin(\theta) = 47.3 e^{-j89.78^\circ} \approx -47.3 j \text{ kV/m} \\ \vec{E}_\varphi = 0 \end{cases}$$

$$\vec{H} = \begin{cases} \vec{H}_r = 0 \\ \vec{H}_\theta = 0 \\ \vec{H}_\varphi = j \frac{k I_0 L}{4\pi} \left(1 + \frac{1}{jk\pi}\right) \frac{e^{-jk\pi}}{\pi} \sin(\theta) = 10.59 e^{-j95.169 \times 10^{-6} j} \approx 10.59 \text{ A/m} \end{cases}$$

$\pi_2 = 20 \text{ m}$

 $k\pi \gg 1$ Comp. distante

$$\vec{E} = \begin{cases} \vec{E}_r = 0 \\ \vec{E}_\theta = j \frac{Z_0 I_0 L}{2\lambda} \cdot \frac{e^{-jk\pi}}{\pi} = 0.94 e^{-j50^\circ} \text{ V/m} \\ \vec{E}_\varphi = 0 \end{cases}$$

$$\vec{H} = \begin{cases} \vec{H}_r = 0 \\ \vec{H}_\theta = 0 \\ \vec{H}_\varphi = \frac{\vec{E}_\theta}{Z_0} = 2.5 e^{-j50^\circ} \text{ mA/m} \end{cases}$$

$$b) \pi = \pi_1 : \frac{|E_\theta|}{|E_\pi|} \approx \frac{1}{2} \quad \frac{|E_\theta|}{|H_\varphi|} \approx 4468$$

$$\pi = \pi_2 : \frac{|E_\theta|}{|E_\pi|} = \infty \quad \frac{|E_\theta|}{|H_\varphi|} = Z_0$$

c) Na zona próxima Na zona distante

$\theta = 90^\circ$

$\theta = 0^\circ$

$H_\varphi \text{ e } E_\theta \text{ e } E_\pi$

$H_\varphi \text{ e } E_\theta$

d) $\pi_1 :$

$$\vec{S}_{av} = \frac{1}{2} Z_0 |H|^2 \hat{n} = 21.14 \hat{n} \text{ kW/m}^2$$

$\pi_2 :$

$$\vec{S}_{av} = \frac{1}{2} Z_0 |H|^2 \hat{n} = 1.17 \text{ mW/m}^2$$

$$d) R_{\pi} = \frac{2\pi}{3} Z \left(\frac{\lambda}{\lambda} \right)^2 = 78.96 \text{ m}\Omega$$

$$P_{\pi} = \frac{1}{2} R_{\pi} I_0^2 = 3.95 \text{ W}$$

5. a) $f = 1 \text{ MHz} \Rightarrow \lambda = 300 \text{ m}$
 $L = 50 \text{ m}$

• $\pi \gg \lambda \Rightarrow \pi > 3 \text{ km} !$
 • $\pi \gg L \Rightarrow \pi > 500 \text{ m}$
 • $\pi > \frac{L^2}{\lambda} \Rightarrow \pi > \frac{50^2}{3} \text{ m}$

e) $f = 10 \text{ GHz} \Rightarrow \lambda = 0.03 \text{ m}$
 $L = 305 \text{ m}$

• $\pi \gg \lambda \Rightarrow \pi > 0.3 \text{ m}$
 • $\pi \gg L \Rightarrow \pi > 3.05 \text{ km}$
 • $\pi > \frac{L^2}{\lambda} \Rightarrow \pi > 6202 \text{ km} !$

b) $f = 100 \text{ MHz} \Rightarrow \lambda = 3 \text{ m}$
 $L = \frac{\lambda}{2} = \frac{3}{2}$

• $\pi \gg \lambda \Rightarrow \pi > 30 \text{ m} !$
 • $\pi \gg L \Rightarrow \pi > 15 \text{ m}$
 • $\pi > \frac{L^2}{\lambda} \Rightarrow \pi > 15 \text{ m}$

f) $\vec{E}_\phi = \vec{E}_r = \vec{H}_\phi = \vec{H}_r = 0$
 $\vec{E}_\theta = Z_0 \vec{H}_\phi$

c) $f = 2.5 \text{ GHz} \Rightarrow \lambda = 0.12 \text{ m}$
 $L = \sqrt{2} \cdot 4 \text{ cm} = 0.04 \cdot \sqrt{2} \text{ m}$

• $\pi \gg \lambda \Rightarrow \pi > 1.2 \text{ m} !$
 • $\pi \gg L \Rightarrow \pi > 0.57 \text{ m}$
 • $\pi > \frac{L^2}{\lambda} \Rightarrow \pi > 0.06 \text{ m}$

d) $f = 12 \text{ GHz} \Rightarrow \lambda = 0.025 \text{ m}$
 $L = 0.6 \text{ m}$

• $\pi \gg \lambda \Rightarrow \pi > 0.25 \text{ m}$
 • $\pi \gg L \Rightarrow \pi > 6 \text{ m}$
 • $\pi > \frac{L^2}{\lambda} \Rightarrow \pi > 28.8 \text{ m} !$

6. $L = 15 \text{ cm} = 0.15 \text{ m}$

a) $f = 1 \text{ GHz} \Rightarrow \lambda = 0.3 \text{ m}$

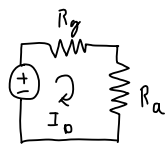
• $\pi \gg \lambda \Rightarrow \pi > 3 \text{ m}$
 • $\pi \gg L \Rightarrow \pi > 1.5 \text{ m}$
 • $\pi > \frac{L^2}{\lambda} \Rightarrow \pi > 0.15 \text{ m}$

b) $R_g = 50 \Omega$

$V_g = 10 \text{ V}$

$Z_a \approx 73.1 \Omega$

$R_p = 0 \Rightarrow R_a = R_L$



$I_0 = \frac{V_g}{R_g + R_a}$

$P_n = \frac{1}{2} R_L |I_0|^2 = 0.2412 \text{ W}$

c) $\vec{E}_\theta = j \frac{Z_0 \vec{I}_m}{2\pi} \cdot \frac{e^{-jkz}}{r} \cdot \frac{\cos(k \frac{L}{2} \cos(\theta)) - \cos(k \frac{L}{2})}{\sin(\theta)}$

$\downarrow L = \frac{\lambda}{2}$

$|\vec{E}| = \left| \frac{Z_0 \vec{I}_m}{2\pi} \frac{\cos(\frac{\pi}{2} \cos(\theta))}{\sin(\theta)} \right|$

$L = \frac{\lambda}{2} \Rightarrow I_0 = -I_m$

$|\vec{E}_{\max}| = \left| \frac{Z_0 I_0}{2\pi} \right|$

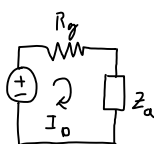
$\vec{S}_{av} = \frac{|\vec{E}_{\max}|^2}{2 Z_0} \hat{r} \Rightarrow \vec{S}_{av_{\max}} = \frac{Z_0 I_0^2}{8 \pi^2 r^2} = 0.315 \text{ mW/m}^2$

e) $L = 1 \text{ cm}$

$R_a = \frac{R_L}{\eta} = \frac{1}{4} \cdot \frac{2\pi}{3} \cdot Z_0 \left(\frac{L}{\lambda}\right)^2 = 0.219 \Omega$

$X_a \approx -Z_L \cot\left(\frac{\pi L}{\lambda}\right) = -3.587 \text{ k}\Omega$

$Z_a = R_a + jX_a$



$I_0 = \frac{V_g}{R_g + Z_a}$

$P_n = \frac{1}{2} R_L |I_0|^2 = 0.8522 \mu\text{W}$

7. a)

$|\vec{E}| = \frac{Z_0 I_0}{\pi} \begin{cases} \cos^2(\theta) & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \text{out} \end{cases}$

$\vec{S}_{av} = |\vec{E}|^2 \cdot \frac{1}{2} \cdot \frac{1}{Z_0} \Rightarrow U = |\vec{E}|^2 \cdot \frac{1}{2} \cdot \frac{1}{Z_0} \cdot \pi^2 = \begin{cases} \frac{I_0^2}{2} \cdot Z_0 \cdot \cos^2(\theta) & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \text{out} \end{cases}$

c) $\alpha_{\text{3dB}} \times Z = \alpha_{\text{3dB}} YZ \text{ p.p. } h \text{ neta}\tilde{\alpha} \tilde{\alpha} \tilde{\alpha}$

$\frac{U}{U_{\max}} = \cos^2(\theta) = \frac{1}{2} \Leftrightarrow \cos(\theta) = \frac{1}{\sqrt{2}} \Leftrightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 23.508$

$\alpha_{\text{3dB}} = 2 \cdot \frac{U}{U_{\max}} = 47.016^\circ$

$D \approx \frac{4\pi}{\alpha_H \alpha_V} \approx 18.66$

b) $P_n = \frac{1}{2} R_L \cdot I_0^2 = \int_0^{2\pi} \int_0^\pi U \sin(\theta) d\theta d\varphi = \frac{I_0^2}{2} \cdot 2\pi \cdot Z_0 \cdot \int_0^{\pi/2} \cos^2(\theta) \cdot \sin(\theta) d\theta = \frac{\pi Z_0 I_0^2}{9}$

$\frac{1}{2} R_L \cdot I_0^2 = \frac{\pi Z_0 I_0^2}{9} \Leftrightarrow R_L = \frac{2\pi}{9} Z_0 = \frac{30}{9} \pi^2 \Omega$

d)

$g(\theta, \varphi) = \frac{4\pi U(\theta, \varphi)}{P_n} = \begin{cases} \frac{\frac{I_0^2}{2} \cdot Z_0 \cdot \cos^2(\theta) \cdot 4\pi}{\frac{\pi Z_0 I_0^2}{9}} & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \text{out} \end{cases} = \begin{cases} 18 \cos^2(\theta) & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \text{out} \end{cases}$

$g_{\max} = D = 18$

8. a) $L = 3 \frac{\lambda}{2}$

$P_n = 2 \cdot 0.4 = 1.8 \text{ W}$

$f = 300 \text{ MHz}$

$P_{in} = 2 \text{ W}$

$\eta = 0.9$

b) $P_{in} = \frac{1}{2} R_a |I_0|^2 \Leftrightarrow |I_0| = \sqrt{\frac{2P_{in}}{R_a}} = 0.195 \text{ A}$

$R_a = 105 \Omega$

$P_{in} = 2 \text{ W}$

c) $d = 100 \text{ m}$

$Z_D ? \Rightarrow PV!$

$\lambda_0 = 1 \text{ m} \Rightarrow \pi \gg \lambda_0 \text{ PV}$

$L = \frac{3}{2} \text{ m} \Rightarrow \pi \gg L \text{ PV}$

$\frac{L^2}{\lambda} = \frac{9}{2} \text{ m} \Rightarrow \pi > \frac{L^2}{\lambda} \text{ PV}$

$\vec{H} = j \cdot \frac{I_m}{2\pi} \cdot \frac{e^{-jkz}}{d} \cdot \frac{\cos(k \frac{L}{2} \cos(\theta)) - \cos(k \frac{L}{2})}{\sin(\theta)} \cdot \hat{\varphi}$

$I_0 = I_m \sin(k_0 \frac{L}{2}) \Rightarrow I_0 = -I_m$

$\vec{H} = j \frac{-I_0}{2\pi} \cdot \frac{e^{-jkz}}{d} \cdot (\dots) = j 366.64 \mu\text{A/m}$

q. DEH

D $\frac{\lambda}{2}$

b) $n = 200 \text{ km} \Rightarrow 2D$

$$|E_o| = \left| \frac{z \bar{I}_m}{2\pi n} \right| = \left| \frac{z \bar{I}_o}{2\pi n} \right| \Rightarrow |I_o| = \frac{2\pi n |E|}{z}$$

$$\bar{I}_m = -I_o$$

a) $R_n = \frac{2\pi}{3} z \left(\frac{L}{\lambda} \right)^2 = 315,8 \text{ m}\Omega$

$$R_p = 2,84 \text{ }\Omega$$

$$R_n = 73,1 \text{ }\Omega$$

$$R_p = R_n \left(\frac{1}{\eta} - 1 \right) = 8,12 \text{ }\Omega$$

$$P_{im} = \frac{1}{2} R_A |I_o|^2 = \frac{1}{2} R_A \left(\frac{2 |E_o| \lambda \pi}{z L} \right)^2 = 4,4 \text{ KW}$$

$$P_{im} = \frac{1}{2} R_A |I_o|^2 = \frac{1}{2} R_A \left(\frac{2\pi n}{z} |E| \right)^2 = 451,23 \text{ W}$$

b) $R_{DEH} \ll R_{\frac{\lambda}{2}} \quad R_{pDEH} \ll R_{p\frac{\lambda}{2}}$

$$P_{RDEH} \gg P_{R\frac{\lambda}{2}} \Rightarrow G_{DEH} \text{ precisa de mais potência para o mesmo campo}$$

$$\alpha_{3dB DEH} > \alpha_{3dB \frac{\lambda}{2}} \Rightarrow \frac{\lambda}{2} \text{ é mais direcional o que é ótimo nas ligações ponto a ponto}$$

$$D_{DEH} < D_{\frac{\lambda}{2}}$$

$$G_{DEH} < G_{\frac{\lambda}{2}} \Rightarrow DEH \text{ tem um ganho menor!}$$

c) $\frac{U}{U_{max}} = \frac{|E|^2}{|E_{max}|^2} = \frac{1}{2}$
 $\Downarrow$
 $\sin(\theta) = \frac{1}{\sqrt{2}}$

$$\alpha_{-3dB} = 90^\circ$$

$$\frac{U}{U_{max}} = \frac{|E|^2}{|E_{max}|^2} = \frac{1}{2}$$

$$\Downarrow$$

$$\frac{\cos(k \frac{L}{2} \cos(\theta)) - \cos(k \frac{L}{2})}{\sin(\theta)} = \frac{1}{\sqrt{2}}$$

$$\frac{\cos(\frac{\pi}{2} \cos(\theta))}{\sin(\theta)} = \frac{1}{\sqrt{2}} \quad \left\{ \begin{array}{l} \theta_1 = 0,8894 \approx 50,96^\circ \\ \theta_2 = 2,2521 \approx 129,04^\circ \end{array} \right.$$

$$\alpha_{3dB} = \theta_2 - \theta_1 = 78,08^\circ$$

d) $D = 4\pi \frac{U_{max}}{P_n} = 4\pi \cdot \frac{\frac{1}{8}}{\frac{\pi}{3}} = \frac{3}{2} = 1,5$

$$P_p = \frac{\pi}{3} z \left(\frac{L}{\lambda} \right)^2 |I_o|^2$$

$$U_{max} = \frac{|E_{max}|^2 \cdot \pi^2}{2 z} = \frac{|z I_o L|^2 \cdot \pi^2}{2 \lambda \pi} = \frac{z}{8} \left(\frac{L}{\lambda} \right)^2 |I_o|^2$$

$$G = \eta D = 0,15$$

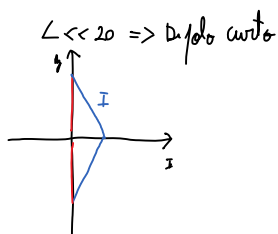
$$D = \frac{4\pi U_{max}}{P_n} = 4\pi \cdot \frac{\frac{z}{8\pi^2} I_o^2}{\frac{1}{2} R_n |I_o|^2} = \frac{z}{\pi R_n} = 1,641$$

$$U_{max} = \frac{|E_{max}|^2 \cdot \pi^2}{2 z} = \frac{z}{8\pi^2} I_o^2$$

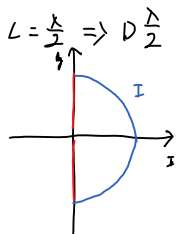
$$P_n = \frac{1}{2} R_n |I_o|^2$$

$$G = \eta D = 1,48$$

10. a) 1: $f_1 = 15 \text{ MHz} \Rightarrow \lambda = 20 \text{ m}$



2: $f_2 = 160 \text{ MHz} \Rightarrow \lambda = 2 \text{ m}$



b) 1: $\frac{U}{U_{max}} = 0$

$\hookrightarrow$ Apenas interferência o fator direcional

$$\left(\frac{\cos(k \frac{L}{2} \cos(\theta)) - \cos(k \frac{L}{2})}{\sin(\theta)} \right)^2 = 0$$

$\Downarrow L \ll \lambda$
 $\sin^2(\theta) = 0 \Leftrightarrow \sin(\theta) = 0 \Rightarrow \left. \begin{array}{l} \theta' = 0 \\ \theta'' = \pi \end{array} \right\}$

$$FNBW = \theta'' - \theta' = 180^\circ$$

2: $\frac{U}{U_{max}} = 0$

$$\left(\frac{\cos(\frac{\pi}{2} \cos(\theta))}{\sin(\theta)} \right)^2 = 0 \Leftrightarrow \cos(\frac{\pi}{2} \cos(\theta)) = 0 \wedge \theta \neq \pi k, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi}{2} \cos(\theta) = \frac{\pi}{2} + \pi s \wedge \theta \neq \pi k; k, s \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow \cos(\theta) = 1 + 2s \wedge \theta \neq \pi k; k, s \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow \theta = \cos^{-1}(1 + 2s) \wedge \theta \neq \pi k; k, s \in \mathbb{Z}$$

As únicas soluções possíveis são π e 0 !

$$\lim_{\theta \rightarrow 0} \frac{\cos(\frac{\pi}{2} \cos(\theta))}{\sin(\theta)} = 0 \quad \text{e} \quad \lim_{\theta \rightarrow \pi} \frac{\cos(\frac{\pi}{2} \cos(\theta))}{\sin(\theta)} = 0$$

$$\text{Logo FNBW} = \pi - 0 = 180^\circ!$$

c) 1: $P_p = \int_0^{2\pi} \int_0^\pi \frac{|E|^2}{2z} \cdot \pi^2 \sin^3(\theta) d\theta d\varphi =$

$$= 2\pi \cdot \frac{\pi}{3} \cdot \frac{|E|^2}{2z} \cdot \pi^2 = \frac{4\pi |E|^2}{3z} \pi^2$$

$$U_{max} = \pi^2 \cdot \frac{|E|^2}{2z}$$

$$D = \frac{4\pi \cdot \pi^2 \cdot \frac{|E|^2}{2z}}{\frac{4\pi |E|^2}{3z} \pi^2} = \frac{3}{2} = 1,5$$

2: $P_n = \frac{1}{2} R_n \cdot I_M^2$

$$U_M = \frac{I_M^2 \cdot z}{8\pi^2}$$

$$D = \frac{4\pi \frac{I_M^2 \cdot z}{8\pi^2}}{\frac{1}{2} R_n I_M^2} = \frac{z}{\pi R_n} = 1,64$$

d) $|E| = 100 \text{ mV/m}$

$$n = 10 \text{ km}$$

1: $P_n = \frac{4\pi |E|^2}{3z} \cdot \pi^2 = 11,11 \text{ KW}$

2: $|E| = \frac{z |I_M|}{2\pi n} = \frac{z |I_o|}{2\pi n} \Rightarrow |I_o| = \frac{|E| \cdot 2\pi \cdot n}{z}$

$$I_M = -I_o$$

$$P_n = \frac{1}{2} \cdot R_n \cdot \left(\frac{|E| \cdot 2\pi \cdot n}{z} \right)^2 = 10,15 \text{ KW}$$

$\circledast \frac{\cos(k \frac{L}{2} \cos(\theta)) - \cos(k \frac{L}{2})}{\sin(\theta)} = \frac{\cos(\pi \frac{L}{\lambda} \cos(\theta)) - \cos(\pi \frac{L}{\lambda})}{\sin(\theta)} \Rightarrow$

Para simplificar da Taylor e como $\frac{L}{\lambda} \ll 1$

$$\Rightarrow \frac{1 - \frac{(\pi \frac{L}{\lambda} \cos(\theta))^2}{2} - (1 - \frac{(\pi \frac{L}{\lambda})^2}{2})}{\sin(\theta)} = \frac{-\frac{(\pi \frac{L}{\lambda} \cos(\theta))^2}{2} + \frac{(\pi \frac{L}{\lambda})^2}{2}}{\sin(\theta)} =$$

$$= \frac{(\pi \frac{L}{\lambda})^2}{2} \cdot \frac{(1 - \cos^2(\theta))}{\sin(\theta)} = \frac{(\pi \frac{L}{\lambda})^2}{2} \cdot \frac{\sin(\theta)}{\sin(\theta)}!$$

f) $dE_{DEH} = j \frac{z K I}{4\pi} \frac{e^{-jk n'}}{n'} \sin(\theta) d\varphi$

$$n' = \sqrt{n^2 + y^2 - 2\pi n y \cos(\theta)} \approx n - y \cos(\theta)$$

$$E_{z0} = j \frac{I_m z k \sin(\theta)}{4\pi n} e^{-jk n'} \int_{-\frac{L}{2}}^{\frac{L}{2}} \sin(k(\frac{L}{2} - n y)) e^{jk n \cos(\theta)} dy =$$

$$= j \frac{z \bar{I}_m}{2\pi} \cdot \frac{e^{-jk n}}{n} \cdot \frac{\cos(k \frac{L}{2} \cos(\theta)) - \cos(k \frac{L}{2})}{\sin(\theta)}$$