Exercise 1

Consider an electron subject to a static magnetic field $B_0 = 1\,T$ along $\vec{z}$ and a resonant magnetic field of amplitude $B_1 = 1\,mT$ along $\vec{x}$ coupled to the electron. After calculating the frequency of the resonant field, derive:

a) the pulse envelope amplitude to operate an $\hat{X}$ gate, when the pulse duration is $t_1 = 100\,ns$.
b) the pulse duration to operate an $\hat{X}$ gate, when the pulse envelope amplitude is $\eta = 0.5$.

$\mu_B = 9.274 \times 10^{-24}\ J/T$

$\hbar = 1.055 \times 10^{-34}\ J \cdot s$

$\eta \Leftrightarrow \Omega$

$\delta \Leftrightarrow$ rotation axis

$t \Leftrightarrow \theta$

$B_0 = 1T \quad \vec{z}$

$B_1 = 1\,mT \quad \vec{x}$

$$\omega_0 = \frac{2\mu_B B_0}{\hbar} = 2\pi \cdot 28\,GHz$$

$$\Omega_0 = \frac{2\mu_B B_1}{2\hbar} = 2\pi \cdot 14\,MHz$$

$$\theta(t) = \int_0^t \eta(t)\,\Omega_0\,dt$$

For square wave:

$$\theta = \eta\,\Omega_0 t$$

a) $t_1 = 100\,ms$

$\hat{X}$ gate $\rightarrow$ Rotation of π around x

$$\theta = \eta\,\Omega_0 t \Rightarrow \eta = \frac{\theta}{\Omega_0 t} = 0.357$$

b) $\eta = 0.5$

$$\theta = \eta\,\Omega_0 t \Leftrightarrow t = \frac{\theta}{\eta\,\Omega} = 71.48\,ms$$

Exercise 2

Consider an electron subject to a static magnetic field $B_0 = 1\,T$ along $\vec{z}$. A resonant magnetic field with tunable phase of amplitude $B_1 = 1\,mT$ along $\vec{x}$ is coupled to the electron by an on/off switch with no amplitude modulation. Calculate the frequency of the resonant field and draw the pulse schedule to operate the gate $\hat{O} = \hat{H} \cdot \hat{X}$.

$B_0 = 1T \quad \vec{z}$

$B_1 = 1\,mT \quad \vec{x}$

$\Omega_0 = 2\pi \cdot 14\,MHz$

On/Off $\Rightarrow \eta = 1$ or 0

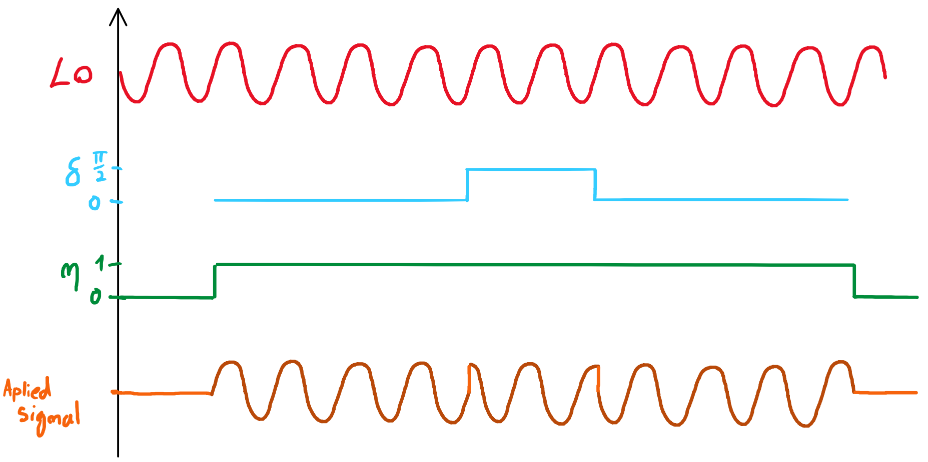
$\hat{O} = \hat{H} \cdot \hat{X} = \hat{X} \cdot \hat{Y}^{1/2} \cdot \hat{X}$

$\hat{H} = \hat{X} \cdot \hat{Y}^{1/2}$

$\hat{X}$: $\delta = 0 \quad \theta = \pi$

$$\theta = \Omega_0 t \Leftrightarrow t = \frac{\theta}{\Omega_0} = 35.73\,ms$$

$\hat{Y}^{1/2}$: $\delta = \frac{\pi}{2} \quad \theta = \frac{\pi}{2} \Rightarrow t = 17.87\,ms$



Exercise 3

An electron is immersed in a static magnetic field $B = 1\,T$ directed along $\vec{z}$ and coupled to a resonant magnetic field of amplitude $B_1 = 1\,mT$ along $\vec{x}$. Calculate the timing accuracy of a control system to provide a rotation angle accuracy $\Delta\theta = \frac{\pi}{1000}$ along $\vec{x}$.

$B_0 = 1\,T \quad \vec{z}$

$B_1 = 1\,mT \quad \vec{x}$

$\Delta\theta = \frac{\pi}{1000}$

$$\Delta\theta = \Omega_0 \Delta t \Leftrightarrow \Delta t = \frac{\Delta\theta}{\Omega_0} = 35.7\,ps$$

$$\Omega_0 = \frac{\mu_B B_1}{\hbar} = 2\pi \cdot 14\,MHz$$

Exercise 1

Consider a state $|\psi\rangle$. Knowing that $P_0(|\psi\rangle) = 0.8, P_1(|\psi\rangle) = 0.3, P_0(H|\psi\rangle) = 0.6, P_1(H|\psi\rangle) = 0.4, P_0(HS^\dagger|\psi\rangle) = 0.7, P_1(HS^\dagger|\psi\rangle) = 0.3$, estimate the angles θ, ϕ localizing the state on the Bloch sphere.

$$P_0(|\psi\rangle) - P_1(|\psi\rangle) = \cos(\theta) \Leftrightarrow \cos(\theta) = 0.5 \Leftrightarrow \theta = \frac{\pi}{3}$$

$$P_0(H|\psi\rangle) - P_1(H|\psi\rangle) = \sin(\theta)\cos(\phi) \Leftrightarrow \frac{\sqrt{3}}{2}\cos(\phi) = 0.2 \Leftrightarrow \cos(\phi) = \frac{4}{10\sqrt{3}}$$

$$P_0(HS^\dagger|\psi\rangle) - P_1(HS^\dagger|\psi\rangle) = \sin(\theta)\sin(\phi) \Leftrightarrow \frac{\sqrt{3}}{2}\sin(\phi) = 0.4 \Leftrightarrow \sin(\phi) = \frac{8}{10\sqrt{3}}$$

$$\tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = 2 \qquad \phi = 1.107\text{ rad}$$

$$63.4^\circ$$

Exercise 2

Derive the pulse schedule to operate $\hat{O} = \hat{X}\hat{H}$ for a spin-qubit quantum computing system equipped with the following primitives:

- $R_z(\theta)$ by free Larmor precession induced by a static field $B_z = 0.5\,T$
- $R_x(\theta)$ by ESR induced by a resonant oscillating field $B_x = 2.7\,mT$ with constant phase

$\hat{O} = \hat{X} \cdot \hat{H}$

$\hat{H} = \hat{z}^{1/2} \cdot \hat{x}^{1/2} \cdot \hat{z}^{1/2}$

$B_z = 0.5\,t$

$$\omega_z = \frac{2\mu_B B_z}{\hbar} = 2\pi \cdot 14\,GHz$$

$B_x = 2.7\,mT$

$$\Omega = \frac{\mu_B B_x}{\hbar} = 2\pi \cdot 38\,MHz$$

$\hat{z}^{1/2}: \frac{\pi}{2} = \omega_z t_z \Leftrightarrow t_z = 17.85\,ps$

$\hat{x}: \pi = \Omega t_x \Leftrightarrow t_x = 13.23\,ns$

$\hat{x}^{1/2}: \frac{\pi}{2} = \Omega t_{x/2} \Leftrightarrow t_{x/2} = \frac{t_x}{2} = 6.618\,ns$

Apply B_z for t_z, apply B_x for $t_{x/2}$, apply $\hat{z}^{1/2}$ again and then apply B_x for t_x

We also need to compensate for the free Larmor precession during the Rabi precession!

$$t_{comp} = \frac{(2\pi - t_x\omega)}{\omega} = \frac{2\pi}{\omega} - t_x$$

Exercise 3

Consider the operator in Exercise 2. Derive the pulse schedule to operate $\hat{O}$ in a spin-qubit quantum computing system equipped with:

- Calibrated $R_x\left(\frac{\pi}{2}\right)$ pulses realized by ESR with $\Omega = 18.896\,MHz$, $\omega = 7\,GHz$
- Virtual-Z gates with arbitrary phase angle θ

$$\lambda = 2\pi - \frac{t_x}{2}\omega$$

Apply Virtual-Z gate for $\frac{\pi}{2}$, apply $R_x(\frac{\pi}{2})$, apply Z gate for $\frac{\pi}{2}$ apply $R(\frac{\pi}{2})$ twice!

In short: $VZ(\lambda)R_x(\frac{\pi}{2}) \cdot VZ(\lambda)R_x(\frac{\pi}{2}) \cdot VZ(\lambda + \frac{\pi}{2}) \cdot R_x(\frac{\pi}{2}) \cdot VZ(\frac{\pi}{2})$

Exercise 4

Consider a quantum system with natural frequency $\omega_{01} = 2\pi \cdot 5\ GHz$ and anharmonicity $\Delta\omega = 2\pi \cdot 200\ MHz$ where a π-rotation pulse along $\vec{x}$ is operated by ESR. Compare the driving amplitudes η and spectral amplitudes $\mathcal{B}$ of the driving field for the $|1\rangle \rightarrow |2\rangle$ transition for a driving pulse with:

a) rectangular envelope of amplitude η_1 and width $t_1 = 10\ ns$.
b) rectangular envelope of amplitude η_2 and width $t_2 = 100\ ns$.
c) Gaussian envelope of amplitude η_3 with FWHM $\Delta t = 10\ ns$.

$\omega_{01} = 2\pi \cdot 5\ GHz$

$\Delta\omega = 2\pi \cdot 200 MHz$

$\omega_{12} = \omega_{01} + \Delta\omega = 2\pi \cdot 5.2\ GHz$

$B(t) = \eta(t) \cdot B_1 \cos(\omega_{01} t)$

$\mathcal{F}(B(t)) = B_1 \delta(\omega_{01}) \mathcal{F}(\eta(t))$

Signal 1:

$$\eta(t) = \eta_1 \cdot \mathrm{rect}\left(\frac{t}{t_1}\right)$$

$$\mathcal{F}(\eta(t)) = \eta_1 \cdot t_1 \cdot \mathrm{sinc}\left(\omega \frac{t_1}{2}\right) \Rightarrow S_1(\omega) = \mathcal{F}(B(t)) = \eta_1 \cdot t_1 \cdot \mathrm{sinc}\left(\frac{(\omega - \omega_{01}) t_1}{2}\right)$$

Signal 2:

$$\eta(t) = \eta_2 \cdot \mathrm{rect}\left(\frac{t}{t_2}\right)$$

$$S_2(\omega) = \eta_2 \cdot t_2 \cdot \mathrm{sinc}\left(\frac{(\omega - \omega_{01}) t_1}{2}\right)$$

Signal 3:

FWHM $\Delta t = 10\ ms \Rightarrow \sigma = \frac{\Delta t}{2.355} = 4.25\ ms$

$$\eta(t) = \eta_3 \cdot e^{-t^2/2\sigma^2}$$

$$S_3(\omega) = \eta_3 \cdot \sqrt{2\pi} \cdot \sigma \cdot B_1 \cdot e^{-\sigma^2(\omega - \omega_{01})^2}$$

$$\frac{S_1(\omega_{01})}{S_1(\omega_{12})} = \mathrm{sinc}\left(\frac{\Delta\omega t_1}{2}\right) = 0.039 \qquad \frac{S_2(\omega_{01})}{S_2(\omega_{12})} = \mathrm{sinc}\left(\frac{\Delta\omega t_2}{2}\right) = 0.0026 \qquad \frac{S_3(\omega_{01})}{S_3(\omega_{12})} = e^{-\sigma^2(\Delta\omega)^2} = 6.56 \cdot 10^{-7}$$